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GRAVITATIONAL WAVES

Gravitational Waves and Papapetrou Metric in the Six-Dimensional Treatment of Gravitation

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Abstract—Six-dimensional treatment of gravitation based on the principle of simplicity to which there corresponds motion of particles with the speed of light in the Compton neighborhood of the three-dimensional space along the geodesics complying with the Fermat principle is given to the Papapetrou metric and gravitational waves. The envelope of the geodesics has the form of a tubular surface with the Compton transverse sizes in the additional subspace where the radius and speed of light vary along the tube. Gravitational waves, which are perturbations of these radii and speed of light, turn out to attenuate exponentially here. Their amplitudes are considered in the near-field zone of the rotator with n Maltese cross lobes and calculated at $n=4$.

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1. INTRODUCTION. STATUS OF THE PAPAPETROU METRIC AND FORMULATION OF THE PROBLEM

It is generally taken that the gravitational field of a nonrotating spherically symmetrical body is governed by a single factor which is the mass of the attracting body. This assertion can be treated as a philosophical principle because it is impossible to prove logically that there are no other governing factors. On the basis of this principle, the gravitational theories where gravitational attraction nonmonotonically varies as the observation point approaches the attracting body have to be thought of as unphysical. Nevertheless, A. Papapetrou in his scalar gravitational theory [1–3] obtained a remarkable metric for the static spherically symmetrical field in isotropic spatial coordinates and the attracting point mass on the assumption that the product of the metric coefficients of the time and space parts is equal to unity while the components of the Ricci tensor were set equal to zero. In the post-Newtonian approximation it coincides with the Schwarzschild metric in the isotropic coordinates and is free of the potential singularity (absence of black holes) at $r/r_g \to 0$. However, the gravitational acceleration and the attracting force for this metric vary nonmonotonically with the radial coordinate r. Here $r_g = 2GM/c^2$ is the gravitational radius, G is the gravitation constant, M is the attracting mass, c is the speed of light at infinity, and the coordinate r is the distance from the center of gravity from the point of view of a distant observer. As r decreases, the gravitational acceleration and the attracting force first increase to their maximum and then smoothly tend to zero (in Fig. 1 the curves depict the gravitational acceleration from the point of view of distant (solid) and local (dotted) observers, the latter is proportional to the attracting force). The nonmonotonic dependence of the attracting force on the radial coordinate in the Papapetrou metric and the fact that possible existence of the second factor also governing the gravitational field is not mentioned in his works were the reasons why the metric was taken to be unphysical. However, the six-dimensional treatment of gravitation under quite different assumptions leads in the spherically symmetrical case exactly to the Papapetrou metric, to two factors governing the gravitational field and gives them a simple physical explanation.

Radii of real bodies in this metric should be larger than a quarter of their gravitational radii, otherwise the gravitational forces inside the body would be smaller than at its lower compression and thus the inner pressure would expand it. Therefore, outside the massive body this metric ensures monotonicity of the gravity force along the radial coordinate and there is no reason to consider it unphysical.

Next, gravitational waves are treated as variations over time and space of the radii of the tubular envelope (of Compton transverse sizes in the additional subspace) of the trajectories of elementary particles in six-dimensional space (we will call it the motion tube). It is shown that application of the Fermat

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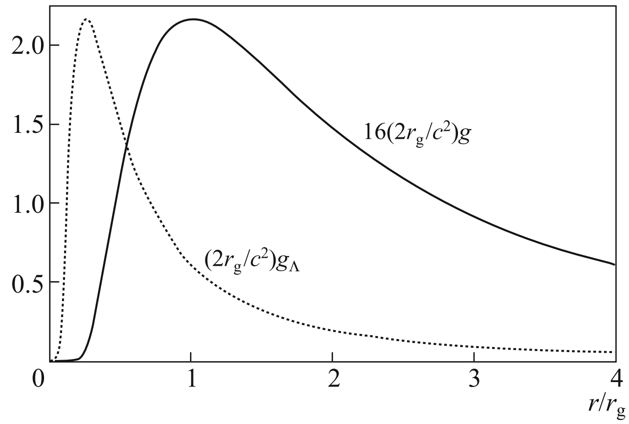

Fig. 1. The solid curve is the gravitational acceleration from the point of view of the distant observer; the dotted curve is the gravitational acceleration proportional to the gravitational force from the point of view of the local observer.

principle to this trajectories results in the Papapetrou metric. Calculation is given for the experiment with gravitational waves in the near field of a rotator.

Note that the dispersion equation is identical both for the acoustic and the electromagnetic waveguide and for the de Broglie waves: $v_{ph}v_g = c^2$, where v_{ph} is the phase velocity of waves, v_g is the group velocity, which is equal to the particle velocity for de Broglie waves, and c is the velocity of waves in an infinite medium. The finite transverse dimension of the waveguide causes dispersion of waves. This indicates that the part of space which we deal with in the experiment is only approximately three-dimensional and has rather small (Compton) sizes in the additional subspace.

2. SIX-DIMENSIONAL TREATMENT OF GRAVITY

Six-dimensional treatment of physics and cosmology [4–9], including gravity, is based on the principle of simplicity [10]. Well fitting into it are Einstein's statement that "Nature saves on principles" and the assumption that the main properties of substance and light are identical, as exemplified by diffraction of electrons and photoeffect. This assumption goes back to F. Klein's idea [11, 12] of the motion of particles with the speed of light in multidimensional space where mechanics is represented as quasi-optics. Six-dimensionality of space was first substantiated by di Bartini in [13] where fundamental physics constants were calculated.

The main property of light is that it propagates at the same velocity in any frame of reference in the absence of gravity. If the main properties of substance and light are identical, particles of substance should move at the speed of light, which is only possible in multidimensional space. When events are projected onto a three-dimensional subspace (X), Newton's formulas referenced to six-dimensional Euclidean space (R_6) yield formulas of relativistic mechanics, Lorentz transformations, the interval of the theory of relativity, spin and isospin, intrinsic magnetic moment, de Broglie waves, the fine-structure formula, the Klein–Gordon equation, CPT symmetry, and the quark model of all particles made up of u and d quarks [5–8].

The complete space is assumed to be six-dimensional because, unlike spaces of fewer dimensions, it allows a simple interpretation of spin and isospin. Particles should be confined to it in the small vicinity of the three-dimensional Universe by the forces (of cosmological nature) orthogonal to it; otherwise, no macroscopic bodies would exist. Let a small region of the Universe, which is of interest for description of the gravitational field, be Euclidean subspace X (the curvature of the Universe in this region is ignored). We assume that at the appropriate choice of time, formulas of Newtonian mechanics are applicable to particles moving in the complete space R_6 and the position of the particles is noted by the observer in the projection onto subspace X.

The particle which is stationary in the projection onto X in the inertial frame of reference K moves with the speed of light c and travels, in the simplest case, in a circle within three-dimensional subspace Y complementing X with respect to R_6 while the center of the circle is in X. In any other inertial frame of reference this particle moves in a helical line (curve a in Fig. 2) located on the cylindrical surface (motion tube) in R_6 with its axis belonging to X. The proper time of the particle is assumed to be proportional to the length of its path in Y. This length is proportional to $|\cos\theta|$, where θ is the angle of inclination of the helical line with respect to the tube directrix. If the particle makes one revolution within the proper time τ, for a fixed observer with respect to whom the particle moves along the tube with the velocity $v = c\sin\theta$ this will occur within the time $t = \tau/|\cos\theta|$, where

$$\sin\theta = v/c, \quad \cos\theta = \pm\sqrt{1-(v/c)^2}. \qquad (1)$$

In (1) and the subsequent discussion the plus sign is for the particle rotating about the axis of the tube in the positive direction and the minus sign is for the antiparticle rotating in the opposite direction. Intervals of the proper time of the particle (or antiparticle) $d\tau$ and of the fixed observer's time dt are related by the formula

$$dt = \pm\frac{d\tau}{\cos\theta} = \frac{d\tau}{\sqrt{1-(v/c)^2}}. \qquad (2)$$

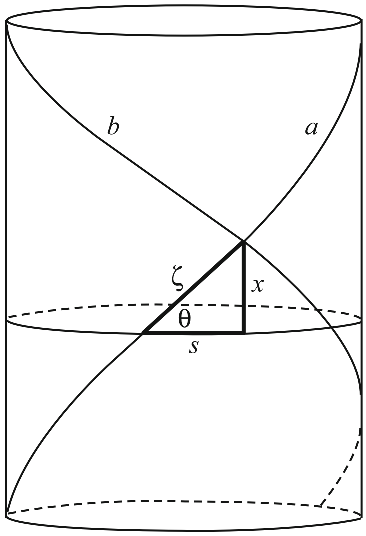

Fig. 2. (a) Helical trajectory of the particle moving with the speed of light c in six-dimensional space over the surface of the cylinder of the Compton radius $a = \hbar/mc$ with the axis in subspace X and the directrix in additional subspace Y. All points of the cylinder surface are at a distance a from X. (b) The helical line of the identical proper time of the particle crossing the trajectory at the right angle and passing through the particle while moving along the cylinder with the velocity of de Broglie waves; its pitch is equal to the de Broglie wavelength.

In the fixed frame K of reference the particle has the component of the velocity $c\cos\theta$ along the directrix. For the fixed observer, the proper time of the particle is, according to (2), also proportional to $\cos\theta$ so that the particle moves with the velocity c in its proper frame K' of reference as well. Displacement of the particle over the interval ds along the motion tube directrix and the corresponding rotation through the angle $d\alpha = ds/a$ about the axis of the tube, where a is the radius of the tube, are identical in any frame of reference. Designating the projection of particle displacement $d\varsigma$ over the surface of the tube onto its generatrix by dx in the system K and using the Pythagorean theorem, we obtain $ds^2 = (cdt)^2 - dx^2$. If this relation is considered as the initial one, it follows from it that $d\varsigma = cdt$, i.e., the particle moves in R_6 with the velocity c.

The particle which is at rest in X moves with the speed of light in Y. Therefore, in Y it has the rest momentum $p_y = mc$, where m is the particle mass, and the rest energy $E = p_y c = mc^2$. By virtue of the principle of similarity of the main properties of substance and light, the rest energy mc^2 should also be equal to $h\nu$, where ν is the frequency of rotation of the particle about the axis of the motion tube. Hence it follows that the radius of the tube is $a = \hbar/mc$ and the length of the directrix is equal to the Compton wavelength, which corresponds to the period h of the coordinate of action in 5-optics [14].

In the gravitational field the radius of the motion tube a and the velocity c_ς of the moving over the tube depend on the position of the particle relative to massive bodies. The metric coefficients depend on the coordinates of subspace X. The relation between a and c_ς is imposed by the condition that the particle moves along the geodesic satisfying the Fermat principle. By definition, $c_\varsigma = d\varsigma/dt$, where $d\varsigma$ is the length of the trajectory segment on the tube which the particle covers within the time dt according to the clock of the distant observer (Fig. 3). We assume that c_ς and a do not depend on the angle θ. In [4] it is shown that

$$(a/c_\varsigma)\cos\theta = \text{const.} \tag{3}$$

Note that all radial directions in any cross section of the motion tube are perpendicular to subspace X and have equal status even if its axis is curvilinear. Therefore, the metric on the tube does not depend on the angular coordinate in the cross section, and the internal geometry [15] on the tube surface is the same as on the corresponding surface of revolution in three-dimensional space.

This treatment is the external geometry of the motion tube suggesting no curvature of space (it is tube rather than space that is deformed). The metric gravitational theory can be treated as the internal geometry of this tube.

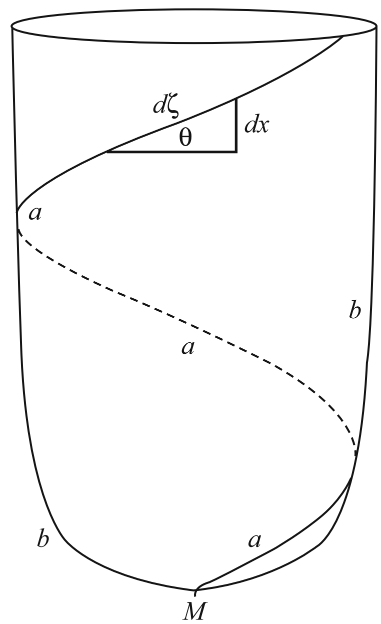

Fig. 3. (a) Trajectory of the particle moving with the speed of light c in six-dimensional space near the point mass M. (b) The envelope of the geodesics—the motion tube of the Compton size.

The projection of the velocity c_ς onto the tangent to the meridian is $v_\varsigma = c_\varsigma \sin\theta$. The coordinate velocity v of the particle recorded by the distant observer is

$$\begin{aligned} v &= \frac{d\sigma}{dt} = v_\varsigma \frac{d\sigma}{d\xi} = c_\varsigma \sin\theta \frac{d\sigma}{d\xi} \\ &= c_\varsigma \sin\theta \big[1 + (\nabla a \cos\beta)^2\big]^{-1/2}, \end{aligned} \tag{4}$$

where ξ and σ are the lengths of the arcs along the meridian and the tube axis, respectively, and β is the angle between ∇a and the tangent to the axis. We obtain the coordinate velocity of light by letting θ in (4) tend to $\pi/2$,

$$c_k = c_\varsigma \frac{d\sigma}{d\xi} = c_\varsigma \big[1 + (\nabla a \cos\beta)^2\big]^{-1/2}. \tag{5}$$

On traveling the length $d\varsigma$ along the tube, the particle rotates through the angle $d\alpha = d\eta/a$, where $d\eta = \cos\theta\, d\varsigma$ is the projection of this traveled length onto the tube directrix. The angle $d\alpha$, identical for any observer, is invariant because the number of revolutions around the tube axis is the same for any observer. The quantity $ds = a_\infty d\alpha$, where a_∞ is the value of the tube radius at infinity, is also invariant. It is the interval in the metric gravitational theory. According to the Pythagorean theorem, $d\eta^2 = d\varsigma^2 - d\xi^2$. Substituting $d\varsigma = c_\varsigma dt$ into this equality, multiplying its both sides by $(a_\infty/a)^2$, taking into account (5), and considering that $d\alpha = d\eta/a$ and $ds = a_\infty d\alpha$, we obtain

$$ds^2 = \gamma(cdt)^2 - \gamma\big[(c/c_k)\, d\sigma\big]^2, \tag{6}$$

where c is the limiting value of the velocity c_ς at infinity and

$$\gamma = (c_\varsigma a_\infty/ca)^2. \tag{7}$$

It follows from (6) that the proper time τ of the particle and the time t of the infinitely distant observer are connected by the relation

$$d\tau/dt = \sqrt{\gamma} \tag{8}$$

and the elements of the spatial distances dl and $d\sigma$ for the local and distant observers, respectively, are connected by the relation

$$dl = \sqrt{\gamma}(c/c_k)\, d\sigma \tag{9}$$

while $ds^2 = (cd\tau)^2 - dl^2$.

Relations (8) and (9) can also be obtained in a different way. For the local observer, the scale of length is the tube radius a (or the Compton wavelength $2\pi a$, which he can measure). He measures lengths along the meridian; consequently,

$$dl = \frac{a_\infty}{a} d\xi = \frac{a_\infty}{a}\frac{d\xi}{d\sigma} d\sigma = \frac{a_\infty c_\varsigma}{a c_k} d\sigma. \tag{10}$$

Hence, in view of the notation (7), we obtain relation (9). For the local observer, the period of revolution of the particle in Y, relative to which it is at rest (in X), serves as the scale of time. This period is proportional to a/c_ς, whence there follows relation (8). According to (7), (8), and (10), the local velocity of the particle is

$$v_\Lambda = \frac{dl}{d\tau} = \frac{a_\infty}{a}\frac{d\xi}{dt}\frac{dt}{d\tau} = \frac{a_\infty}{a} c_\varsigma \sin\theta \frac{1}{\sqrt{\gamma}} = c\sin\theta,$$

and

$$\frac{v}{c_k} = \frac{v_\varsigma}{c_\varsigma} = \frac{v_\Lambda}{c} = \sin\theta. \tag{11}$$

The limiting (as $\sin\theta \to 1$) local velocity is equal to the speed of light at infinity.

In view of (7), relations (3) and (8) can be represented as

$$\frac{1}{\sqrt{\gamma}}\cos\theta = \text{const}, \quad \frac{dt}{d\tau}\cos\theta = \text{const}. \tag{12}$$

Formulas (11) and (12) allow the particle velocity to be expressed in terms of the function γ,

$$\left(\frac{v}{c_k}\right)^2 = \left(\frac{v_\varsigma}{c_\varsigma}\right)^2 = \left(\frac{v_\Lambda}{c}\right)^2 = 1 - \frac{\gamma}{\gamma_0}\cos^2\theta_0 = 1 - \frac{\gamma}{\gamma_0}\left\{1 - \left[\frac{(v_\Lambda)_0}{c}\right]^2\right\}, \tag{13}$$

where the subscript zero is used to label values of quantities at the initial moment of time.

For the local observer, the particle acceleration is $dv_\Lambda/d\tau$. In view of (13), we obtain

$$\frac{dv_\Lambda}{d\tau} = \frac{dv_\Lambda}{dl}\frac{dl}{d\tau} = \frac{1}{2}\frac{dv_\Lambda^2}{dl} = -\frac{c^2}{2}\frac{\cos^2\theta_0}{\gamma_0}\frac{d\gamma}{dl}.$$

Hence the gravitational acceleration for him will be $g_\Lambda = (c^2/2)d\nu/dl_{\|}$, where $\nu = \ln\gamma$ and $dl_{\|}$ is the element of the distance in space along the gradient of the function γ from the point of view of the local observer. Introducing the gravitational potential Φ_Λ through the equality $g_\Lambda = d\Phi_\Lambda/dl_{\|}$ and carrying out integration, we obtain

$$\sqrt{\gamma} = \exp\left(-\frac{1}{c^2}\int_{l_\parallel}^{\infty} g_\Lambda dl_\parallel\right) = \exp\left(\frac{1}{c^2}\Phi_\Lambda\right) = \frac{d\tau}{dt},$$
$$\nu = 2\Phi_\Lambda/c^2. \tag{14}$$

Formula (14) describes slowing-down of time in the gravitational field. Eliminating $\sqrt{\gamma}$ in it through (12) and considering (11), we find that along the geodesic

$$\left[1 - \left(\frac{v_\Lambda}{c}\right)^2\right] \exp\left(-\frac{2}{c^2}\Phi_\Lambda\right) = \text{const.} \tag{15}$$

In a weak field, formulas (14) and (15) are reduced to the form $d\tau/dt = 1 + \Phi_\Lambda/c^2$, $v_\Lambda^2/2 + \Phi_\Lambda = \text{const}$. The latter is the energy conservation law in Newtonian mechanics.

Similarly, the gravitational acceleration from the point of view of the distant observer is $g = c_\parallel^2(d\nu/2d\sigma_\parallel)$, $d/d\sigma_\parallel$ means differentiation along the gradient of the function γ, and $c_\parallel$ is the value of c_k in this direction.

The particle which is at rest in X rotates in Y with the frequency $\nu_0 = c_\varsigma/2\pi a$ while its rest energy is

$$E_0 = h\nu_0 = \frac{\hbar c_\varsigma}{a} = \frac{\hbar\sqrt{\gamma}c}{a_\infty} = mc^2\sqrt{\gamma}.$$

The total energy of the particle moving in X is $E = E_0/|\cos\theta|$. The Lagrange formalism yields the same. Action S is defined to an accuracy of the constant factor as the integral of a scalar. Here the scalar is the angle through which the particle rotates about the axis of the motion tube. We choose the constant factor such that in the absence of gravity the Lagrange function is $L = -mc^2\cos\theta$ as in relativistic mechanics. Then

$$S = -\hbar\int_{\alpha_1}^{\alpha_2} d\alpha = \int_{t_1}^{t_2} L\,dt.$$

Hence we obtain $L = -\hbar\dot{\alpha}$. From (6) we have $\dot{\alpha} = (1/a)\sqrt{c_\varsigma^2 - v_\varsigma^2}$, so that $L = -(\hbar/a)\sqrt{c_\varsigma^2 - v_\varsigma^2} = -(\hbar c/a_\infty)\sqrt{\gamma}\cos\theta$. The energy of the particle and the projection of its momentum onto the meridian of the tube are, respectively,

$$E = p_\xi v_\varsigma - L = \frac{\hbar c_\varsigma}{a\cos\theta} = \frac{\hbar c}{a_\infty}\frac{\sqrt{\gamma}}{\cos\theta} = \frac{mc^2\sqrt{\gamma}}{\cos\theta},$$
$$p_\xi = \frac{\partial L}{\partial v_\varsigma} = \frac{\hbar v_\varsigma}{a\sqrt{c_\varsigma^2 - v_\varsigma^2}} = \frac{\hbar}{a}\tan\theta,$$

and thus $p_\xi = Ev_\varsigma/c_\varsigma^2$ and the total particle momentum $p = \partial L/\partial c_\varsigma$ is $p = E/c_\varsigma = \hbar/(a\cos\theta)$. It is evident from this and from (12) that along the geodesic we have $E = \text{const}$. What occurs here is only flow of the motion energy from its implicit form in subspace Y to the explicit form in subspace X or vice versa. The potential energy is exactly the stored motion energy in Y.

In the absence of gravity, the particle which is at rest in X travels in the circle of radius a_∞ with the speed of light c. The centripetal cosmological force corresponding to this circular motion is $F = p_y c/a_\infty = \hbar c/a_\infty^2 = mc^2/a_\infty$, c^2/ag times larger than the weight of the particle at the Earth's surface, which is 2.38×10^{28} for the electron. The same result is obtained when the particle moves in the helical line: $F = pcK/\cos\theta$, where $K = \cos^2\theta/a_\infty$ is the curvature of the helical line. Here it is evident that $F = \hbar c/a_\infty^2$ at any θ.

The cosmological force has the form of the Lorentz force acting on the charged particle moving in the additional subspace in a magnetic field aligned with the radii of the Universe—an expanding three-dimensional sphere [9] where the mass of the elementary particle plays the role of the charge. Thus, the mass of the elementary particle is its charge with respect to this magnetic field, and the presence of this field is the condition for existence of both microparticles and macroscopic bodies in the three-dimensional Universe.

The main difference of the approach under consideration from other multidimensional theories [16–20] treating the three-dimensional Universe as a 3-brane in the space of higher dimensionality is the presence of a cosmological force confining particles to the Compton neighborhood of the three-dimensional subspace. Existence of this force is not postulated but rather follows from the principle of simplicity, which is rendered here in its particular case as the principle of similarity of the main properties of substance and light, according to which $mc^2 = h\nu$ (whence this force is $p_y c/a_\infty = m^2c^3/\hbar$), and from existence of three-dimensional bodies. If this force did not exist, elementary particles would not be confined to the neighborhood of the three-dimensional subspace. In this approach compactification of additional dimensions of space is replaced by the cosmological force that confines particles to the Compton neighborhood of the three-dimensional subspace. It is trajectories of elementary particles in the additional space than additional dimensions that are compactified here.

In the gravitational field the angle of inclination of the meridian with respect to the tube axis, χ, is defined by the relations $\sin\chi = da/d\xi$, $\cos\chi = \sqrt{1 - (da/d\xi)^2} = 1/\sqrt{1 + (da/d\sigma)^2}$, and $\tan\chi = da/d\sigma$. The component of the cosmological force perpendicular to the geodesic and the tangent plane is equal to the centripetal force proportional to curvature K,

$$\frac{pc_\varsigma K}{\cos\theta} = F\cos\chi, \qquad (16)$$

where $K=\sqrt{K_\perp^2+(\sigma'')^2}$, $K_\perp^2=(y_1'')^2+(y_2'')^2$, y_1 and y_2 are the coordinates of the particle in two mutually perpendicular directions in the tube cross section, and the prime denotes a derivative along the trajectory. These coordinates can be written as $y_1=a\cos\alpha$ and $y_2=a\sin\alpha$. Then from the formulas $ad\alpha=\cos\theta\,d\varsigma$, $d\xi=\sin\theta\,d\varsigma$, $\sigma'=\cos\chi\sin\theta$, and (12) we obtain

$$\begin{aligned}\sigma'' &= -\cos\chi\cos^2\theta_0\frac{1}{2\gamma_0}\frac{d\gamma}{d\xi}\\ &- \frac{1}{\cos\chi}\left(1-\cos^2\theta_0\frac{1}{\gamma_0}\gamma\right)\frac{da}{d\xi}\frac{d^2a}{d\xi^2}, \qquad (17)\\ K_\perp^2 &= (a\alpha'^2-a'')^2+(a\alpha''+2a'\alpha')^2.\end{aligned}$$

Substitution of the expressions obtained for p and F into (16) yields $a_\infty\sqrt{\gamma}K/\cos^2\theta=\cos\chi$; from this and from (17) we obtain that $\sqrt{\gamma}a_\infty/a\approx 1$ accurate to the value $a_\infty^2[(d\sqrt{\gamma}/d\xi)^2+|d^2\sqrt{\gamma}/d\xi^2|]$. By virtue of (7), we have

$$\frac{a}{a_\infty}=\sqrt{\gamma}, \quad \frac{c_\varsigma}{c}=\frac{c_\parallel}{c}=\frac{c_\perp}{c}=\gamma, \qquad (18)$$

where $c_\perp$ is the speed of light in the direction perpendicular to the gradient of the field.

The Lagrange function

$$L=-\hbar\sqrt{1-(\dot r/c_\parallel)^2-(\dot r/c_\perp)^2}$$

in the polar coordinates r,φ does not explicitly depend on φ; thus, $\partial L/\partial\dot\varphi=\text{const}$, whence we obtain the angular momentum conservation law $(c/c_\perp)^2 rv\sin\beta=\text{const}$. Substituting (18) into it, we get $v(r/\gamma)\sin\theta\sin\beta=\text{const}$.

In the six-dimensional treatment of gravity massive bodies themselves do not produce gravity but only decrease the speed of light in their vicinity. As a result, the radius of the orbit in Y decreases while the centrifugal and cosmological forces are equal. In this case the particle motion tube differs from the cylindrical surface, and its meridians become inclined to the axis, which results in that the projection of the cosmological force onto the meridian becomes different from zero. It is $F_\xi=-F\sin\chi=-F\,da/d\xi$ and represents the gravitational force, as is evident from both the equality $p_\xi=p\sin\theta=pv_\varsigma/c_\varsigma$ and the relations for the time derivatives of the projections of particle velocity and momentum onto the meridian at $v=0$:

$$\begin{aligned}\frac{d}{dt}v_\varsigma &= -c_\varsigma^2\frac{1}{\sqrt{\gamma}}\frac{d}{d\xi}\sqrt{\gamma},\\ \frac{d}{dt}p_\xi &= \frac{\hbar}{ac_\varsigma}\frac{dv_\varsigma}{dt} = -\frac{\hbar c_\varsigma}{a\sqrt{\gamma}}\frac{d}{d\xi}\sqrt{\gamma}\\ &= -\frac{\hbar c}{a_\infty}\frac{d}{d\xi}\sqrt{\gamma} = -Fa_\infty\frac{d}{d\xi}\sqrt{\gamma},\end{aligned}$$

which, according to (18), is equal to F_ξ.

In the spherically symmetrical field the asymptotic expansion of the function γ in powers of $1/r$, where r is the radial coordinate (the distance from the center of gravity from the point of view of the distant observer), has the form

$$\gamma = 1-(r_g/r)+b_2(r_g/r)^2+b_3(r_g/r)^3+\ldots \qquad (19)$$

Here $r_g=2GM/c^2$ is the gravitational radius, G is the gravitation constant, and M is the mass of the attracting body. In (19) the coefficient of r_g/r to the first power was chosen to be -1, as in the metric gravitational theory, so that the gravitational potential far from the center of gravity was Newtonian [21, 22].

Gravity acts on the light rays in same manner as the corresponding anisotropic medium, and the speed of light c_k is described by the formula of the ray velocity [23]

$$\frac{1}{c_k^2}=\left(\frac{\sin\beta}{c_\perp}\right)^2+\left(\frac{\cos\beta}{c_\parallel}\right)^2, \qquad (20)$$

where β is the angle between the light propagation direction and the gradient of the field. Designating the projections of the element $d\sigma$ of the trajectory in X onto the directions of the field gradient and onto the direction perpendicular to it by $d\sigma_\parallel$ and $d\sigma_\perp$, respectively, and substituting (20) into (6), we obtain

$$ds^2=\gamma(cdt)^2-\gamma\left(\frac{c}{c_\parallel}d\sigma_\parallel\right)^2-\gamma\left(\frac{c}{c_\perp}d\sigma_\perp\right)^2.$$

Neglecting quantum corrections under conditions (18), we obtain

$$ds^2=\gamma(cdt)^2-\frac{1}{\gamma}d\sigma_\parallel^2-\frac{1}{\gamma}d\sigma_\perp^2. \qquad (21)$$

Metric (21) is described by only one coordinate function, namely, the function γ.

The centrifugal force $p_\xi v_\varsigma/R\cos\theta$, where R is the curvature radius of the trajectory in X, is counterbalanced by the gravitation force component $F_\xi\sin\beta$. Hence, $\tan^2\theta=(R/\sqrt{\gamma})(d\sqrt{\gamma}/d\xi)\sin\beta$.

To introduce coordinates as applied to metric (21), we will use the Einstein equation for the Ricci tensor component R_{00}. In vacuum, $R_{00}=0$. In the static spherically symmetrical field this equation is reduced to the form $\nu''+\nu'(2/r)=0$ at $\gamma=\exp v$

[21]. Its solution satisfying the asymptotic form (19) is $\nu = -r_g/r$, $b_2 = 1/2$ [4]. Its substitution into the above expressions for the gravitational acceleration in empty space results, from the point of view of the distant and local observers, in

$$g = \frac{c^2 r_g}{2r^2}\exp\left(-2\frac{r_g}{r}\right) = \frac{GM}{r^2}\exp\left(-2\frac{r_g}{r}\right),$$
$$g_\Lambda = \frac{c^2 r_g}{2r^2}\exp\left(-\frac{r_g}{2r}\right) = \frac{GM}{r^2}\exp\left(-\frac{r_g}{2r}\right)$$

(Fig. 1). The g reaches its maximum $g = c^2/2r_g e^2 = c^4/4GMe^2$ at $r = r_g$. The functions g and g_Λ tend to zero as $r/r_g \to 0$ because the motion tube radius a and accordingly the projection F_ξ of the cosmological force onto the tube meridian tend to zero as well.

Measuring the circumference $2\pi l$ which is drawn from the center of gravity and in which he finds himself, the local observer can obtain r from the formula $l = r/\sqrt{\gamma} = r\exp(r_g/2r)$. At $l = r_g e^2/4$, which corresponds to the radial coordinate $r = r_g/4$, g_Λ reaches the maximum $g_\Lambda = 8c^2/r_g e^2 = 4c^4/GMe^2$. It is important that l has a minimum $l = r_g e/2$ at $r = r_g/2$. In the case of a point mass, if it ever existed, the length l infinitely increases not only as r increases but also as r tends to zero because the radius of the motion tube and the linear scale of the local observer tend to zero.

In the spherically symmetrical field, where $\gamma = \exp\nu$ $(\nu = -r_g/r)$, metric (21) turns out to be the Papapetrou metric and coincides in the post-Newtonian approximation with the Schwarzschild metric in isotropic coordinates [21, 22] and with the metric of the relativistic gravitation theory [24] but differs from them in the next approximation. However, in the static case the radius of the massive body cannot be smaller than a quarter of its gravitational radius from the point of view of the distant observer.

This solution is also obtained from summation of partial local gravitational potentials ν_j for any, including infinitesimal, components M_j of the total mass M, so that $\nu = \sum_j \nu_j$. Indeed, for $M_j = M/n$ $(r_{gj} = r_g/n)$ we have $\nu_j = -r_{gj}/r$, $\nu = \lim_{n\to\infty}(n\nu_j) = -r_g/r$. This summation agrees with the principle of simplicity at the spatial mass distribution as well. In any case, for this static mass distribution, the replacement of the exponent $\gamma = \exp[-\sum_j (r_{gj}/r_j)]$ by the first three terms of the series expansion yields a metric coinciding with the principal terms of the expansion in the post-Newtonian metric given in [22].

In this approach the cosmological force confining particles within the Compton neighborhood of three-dimensional space takes the place of compactification of additional space. There are compactified trajectories of elementary particles in an additional space rather than additional dimensions.

3. GRAVITATIONAL WAVES IN THE SIX-DIMENSIONAL TREATMENT OF GRAVITY

To describe the time-variable field, we use the Einstein equation [21] with zero indices for metric (21). In the ordinary representation $ds^2 = g_{ik}dx^i dx^k$, where $x^0 = ct$; metric (21) in the Cartesian coordinates x^1, x^2, x^3 has nonzero covariant coefficients $g_{00} = \gamma$, $g_{11} = g_{22} = g_{33} = -1/\gamma$, and at small v^2/c^2 this equation takes the form

$$R_{00} = -4\pi\frac{G}{c^2}\gamma\left(\rho + \frac{3p}{c^2}\right). \qquad (22)$$

Here ρ is the density of the rest mass and p is the pressure,

$$R_{00} = \frac{\partial}{\partial x^0}\Gamma^\alpha_{\alpha 0} + \Gamma^\alpha_{0\beta}\Gamma^\beta_{\alpha 0} - \frac{\partial}{\partial x^\alpha}\Gamma^\alpha_{00} - \Gamma^\alpha_{\alpha\beta}\Gamma^\beta_{00}, \qquad (23)$$

and the nonzero contravariant coefficients g^{ik} and the Christoffel symbols

$$\Gamma^k_{ij} = \frac{1}{2}g^{kl}\left(\frac{\partial}{\partial x^j}g_{li} + \frac{\partial}{\partial x^i}g_{lj} - \frac{\partial}{\partial x^l}g_{ij}\right)$$

are as follows: $g^{00} = 1/\gamma$, $g^{11} = g^{22} = g^{33} = -\gamma$, $\Gamma^0_{11} = \Gamma^0_{22} = \Gamma^0_{33} = -\gamma^{-2}\nu_0/2$, $\Gamma^0_{00} = -\Gamma^1_{01} = -\Gamma^2_{02} = -\Gamma^3_{03} = \nu_0/2$, $\Gamma^1_{00} = \gamma^2\nu_1/2$, $\Gamma^0_{20} = \Gamma^2_{11} = \Gamma^2_{33} = -\Gamma^1_{21} = -\Gamma^2_{22} = -\Gamma^3_{23} = \nu_2/2$, $\Gamma^2_{00} = \gamma^2\nu_2/2$, $\Gamma^3_{00} = \gamma^2\nu_3/2$, $\Gamma^0_{30} = \Gamma^3_{11} = \Gamma^3_{22} = -\Gamma^1_{31} = -\Gamma^2_{32} = -\Gamma^3_{33} = \Gamma^0_{30} = \nu_3/2$, where ν and γ are connected by the relation $\gamma = \exp\nu$ with the subscript of ν denoting differentiation with respect to the corresponding coordinate. Hence we find $\Gamma^\alpha_{\alpha\beta} = -\nu_\beta$, and (23) is reduced to the form

$$R_{00} = \frac{1}{2}(3\nu_0^2 - 3\nu_{00} - \gamma^2\Delta\nu)$$
$$= \frac{1}{2}\left(\frac{3}{c^2}\dot{\nu}^2 - \frac{3}{c^2}\ddot{\nu} - \gamma^2\Delta\nu\right).$$

In empty space we have

$$\frac{3}{c^2}\dot{\nu}^2 - \frac{3}{c^2}\ddot{\nu} - \gamma^2\Delta\nu = 0,$$

and in the linear approximation it is

$$\frac{3}{c^2}\ddot{\nu} + \gamma^2\Delta\nu = 0. \qquad (24)$$

From (24) and (7) it is evident that gravitational waves are perturbations of both the speed of light and the motion tube radii. Equation (24) differs from the wave equation only by the sign of the Laplacian. This means that the propagation velocity of weak gravitational waves is equal to the imaginary value $-ic/\sqrt{3}$. These waves are analogous to inhomogeneous waves in the waveguide which show exponential attenuation with distance. Solutions to (24) in the weak gravitational field are spherical waves of the form $\nu=-(r_g/r)\exp(-kr-i\omega t)$, where $k=\omega\sqrt{3}/c$. At the frequencies of 1 Hz and 1 kHz the waves are attenuated by the factor e over the distances of 27 565 km and 27.6 km, respectively. This attenuation rules out the possibility of receiving gravitational waves from celestial bodies even as close as the Moon.

A distributed source of spherical waves is the space-time spectral density of the right-hand side of (22), which for weak fields with the pressure ignored in comparison with the proper energy of the unit volume mass is reduced to the form

$$\frac{3}{c^2}\ddot{\nu}+\Delta\nu=\frac{8\pi\rho G}{c^2}. \qquad (25)$$

When the solid body uniformly rotates with the angular velocity ω, we have

$$\rho=\overline{\rho}+\sum_{n\geqslant 1}\left[\rho_{cn}\cos(n\varphi)+\rho_{sn}\sin(n\varphi)\right],$$

where $\overline{\rho}$ is the time-average value for the density of the body, $\varphi=\omega t$, and

$$\rho_{cn}(\mathbf{r}')=\frac{1}{\pi}\int\limits_0^{\pi}\rho(\mathbf{r}',t)\cos(n\varphi)\,d\varphi,$$
$$\rho_{sn}(\mathbf{r}')=\frac{1}{\pi}\int\limits_0^{\pi}\rho(\mathbf{r}',t)\sin(n\varphi)\,d\varphi, \qquad (26)$$

The solution to (25) vanishing at infinity has the form

$$\nu=\overline{\nu}+\sum_{n\geqslant 1}\left[\nu_{cn}\cos(n\omega t)+\nu_{sn}\sin(n\omega t)\right],$$

where

$$\nu_{cn}=-\frac{2G}{c^2}\iiint\limits_V\frac{\exp(-nkR')}{R'}\rho_{cn}(\mathbf{r}')\,dV',$$
$$\nu_{sn}=-\frac{2G}{c^2}\iiint\limits_V\frac{\exp(-nkR')}{R'}\rho_{sn}(\mathbf{r}')\,dV',$$

R' is the distance from the integration element to the observation point, the integration is carried out over the volume of the body. As the source of gravitational waves we consider a rotator with n lobes, its rotation axis is the nth-order symmetry axis and the lobes are shaped as part of a disc of radius L and thickness $h_\perp$ with the material of density ρ uniformly distributed over the region $0\leqslant r'\leqslant \ell$, $0\leqslant z'\leqslant h_\perp$ and the regions $\ell\leqslant r'\leqslant L$, $-\varphi_0\leqslant\varphi'-2\pi m/n\leqslant\varphi_0$ at $m=0, 1, 2, \ldots, n-1$, where r', z', and φ' are the cylindrical coordinates for the disc. For this rotator $\rho(\mathbf{r}',t)$ does not depend on the radial coordinate in the region $\ell\leqslant r'\leqslant L$. At $n=4$ the rotator has the form of a Maltese cross. Integration in (26) yields

$$\rho_{cn}(\varphi')=\rho\frac{2}{\pi}\sin(n\varphi_0)\cos(n\varphi'), \qquad \rho_{sn}(\varphi')=\rho\frac{2}{\pi}\sin(n\varphi_0)\sin(n\varphi'),$$
$$\nu_{cn}=-8G\frac{\rho}{c^2\pi}\sin(n\varphi_0)\int\limits_0^{h_\perp}dz'\int\limits_0^{\pi}\cos(n\varphi')\,d\varphi'\int\limits_{\ell}^{L}\frac{\exp(-nkR')}{R'}r'dr', \qquad \nu_{sn}=0,$$

where $R'=\sqrt{r^2+r'^2-2r'r\cos\varphi'+z'^2}$, r is the distance from the axis to the observation point.

The gravitational acceleration at the nth harmonic is

$$g_n=\frac{c^2}{2}\frac{d\nu_n}{dr}=\frac{4}{\pi}G\rho\sin(n\varphi_0)\int\limits_0^{\pi}\cos(n\varphi')\,d\varphi'\int\limits_0^{h_\perp}dz'\int\limits_{\ell}^{L}\frac{\exp(-nkR')}{R'}\left(nk+\frac{1}{R'}\right)\frac{r-r'\cos\varphi'}{R'}r'dr'.$$

Expanding the integrand in powers of nk, we find, for $(nkr)^4\ll 1$, the difference between g_n and its quasi-stationary value corresponding to $k=0$:

$$\delta g_n=-\frac{2n^2k^2}{\pi}G\rho\sin(n\varphi_0)\int\limits_0^{\pi}\cos(n\varphi')\,d\varphi'\int\limits_0^{h_\perp}dz'\int\limits_{\ell}^{L}\left(\frac{1}{R'}-\frac{2}{3}nk\right)(r-r'\cos\varphi')r'dr'.$$

Integrating first over r' and then over z', we obtain

$$\delta g_n=-\frac{n^2\omega^2G\rho}{\pi c^2}\sin(n\varphi_0)\int\limits_0^{\pi}\left[h_\perp r\chi(\varphi')+\psi_b(\varphi')+\psi_L(\varphi')-\psi_l(\varphi')-\psi_a(\varphi')\right]\cos(n\varphi')\,d\varphi',$$

where

$$\chi(\varphi') = \left(5\sin^2\varphi' - 2 - \frac{L}{r}\cos\varphi'\right)R_L(\varphi') - \left(5\sin^2\varphi' - 2 - \frac{l}{r}\cos\varphi'\right)R_l(\varphi'),$$

$$\psi_b(\varphi') = h_\perp\cos\varphi'(9r^2\sin^2\varphi' + h_\perp^2)\ln\left[\frac{R_L(\varphi') + L - r\cos\varphi'}{R_l(\varphi') + l - r\cos\varphi'}\right],$$

$$\psi_a(\varphi') = 4r^3\sin^2\varphi'\sin(2\varphi')\big[\psi_{aL}(\varphi') - \psi_{al}(\varphi')\big],$$

$$\psi_L(\varphi') = \Big\{r^3\big[4\sin^2\varphi'(2\sin^2\varphi' - 1) - 1\big] + 3rL^2 - 2L^3\cos\varphi'\Big\}\ln\left[\frac{R_L(\varphi') + h_\perp}{R_{L0}(\varphi')}\right],$$

$$\psi_l(\varphi') = \Big\{r^3\big[4\sin^2\varphi'(2\sin^2\varphi' - 1) - 1\big] + 3rl^2 - 2l^3\cos\varphi'\Big\}\ln\left[\frac{R_l(\varphi') + h_\perp}{R_{l0}(\varphi')}\right],$$

$$\psi_{aL}(\varphi') = \arctan\frac{r^2 + L^2 - 2Lr\cos\varphi' + R_L(\varphi')(L - r\cos\varphi')}{h_\perp r\sin\varphi'},$$

$$\psi_{al}(\varphi') = \arctan\frac{r^2 + l^2 - 2lr\cos\varphi' + R_l(\varphi')(l - r\cos\varphi')}{h_\perp r\sin\varphi'},$$

$$R_L(\varphi') = \sqrt{h_\perp^2 + r^2 + L^2 - 2Lr\cos\varphi'}, \qquad R_l(\varphi') = \sqrt{h_\perp^2 + r^2 + l^2 - 2lr\cos\varphi'},$$

$$R_{L0}(\varphi') = \sqrt{r^2 + L^2 - 2Lr\cos\varphi'}, \qquad R_{l0}(\varphi') = \sqrt{r^2 + l^2 - 2lr\cos\varphi'}.$$

The resonant receiver of gravitational waves can be, for instance, a flexurally vibrating horizontal rod aligned perpendicularly to the axis of the system, whose free end is near the rotator.

For the vibration amplitude $A_n(\omega)$ of the linear resonator corresponding to the gravitational acceleration $g_n(\omega)$ we have $A_n(\omega) = g_n(\omega)/(2\beta\sqrt{n^2\omega^2 + \beta^2})$ at the resonant angular frequency $n\omega$, where β is the attenuation coefficient. Considering that $\delta g_n(\omega)/\delta g_n(\omega_1) = \omega^2/\omega_1^2$, where ω_1 is another resonant frequency, we obtain

$$\delta g_n(\omega) = \frac{g_n(\omega) - g_n(\omega_1)}{1 - (\omega_1/\omega)^2} = 2\frac{A_n(\omega)\beta\sqrt{n^2\omega^2 + \beta^2} - A_n(\omega_1)\beta_1\sqrt{n^2\omega_1^2 + \beta_1^2}}{1 - (\omega_1/\omega)^2}.$$

Measuring amplitudes of vibrations at two resonant frequencies, we can find δg_n and δA_n.

If the receivers of gravitational waves are arranged periodically along the azimuth with the period of $2\pi/n$ and are connected in parallel on the electric side of the receiving system, the vibration amplitude of the received signal for the nth harmonic increases by a factor of n. With another group of receivers arranged at the azimuth angles half way among the receivers of the first group and connected to them in antiphase, the total signal additionally increases by a factor of two. In this signal detection scheme at $\rho = 3\,\mathrm{g\,cm^{-3}}$, $L = 2\,\mathrm{m}$, $\ell = 0.3L$, $n\varphi_0 = \pi/3$, $h_\perp = 0.7L$, $r = 2.4\,\mathrm{m}$, $f = \omega/2\pi = 40\,\mathrm{Hz}$, and $\beta = \pi 10^{-5}\,\mathrm{Hz}$ we obtain $8\delta A_4(\omega) = -2.06\times10^{-15}\,\mathrm{cm}$ and $8\delta g_4(\omega) = -1.301\times10^{-16}\,\mathrm{cm\,c^{-2}}$ (to the given value of the coefficient β there corresponds the resonator Q-factor 10^8 at the frequency 1 kHz). Generally speaking, it is reasonable to use a lot of receivers summing their signals on the electric side of the receiving system with a phase shift of $-n\varphi_m$, where φ_m is the azimuth angle of the mth receiver. The amplitudes of the signals are summed in phase, and the noise is summed at random phases.

Alternatively, if the gravitational waves are thought to propagate with the speed of light and superposition of local gravitational potentials is used, we find that δg_n and δA_n will decrease by a factor of three and will have the opposite sign.

In the gravitational wave there is a restoring elastic force, which is linear at small vibrations. Here it is equal to the difference of the centrifugal and cosmological forces

$$\delta F = F - m(c_\varsigma^2/a) = \big(1 - \gamma^{3/2}\big)F = \big[1 - (a/a_\infty)^3\big]F \cong 3\varepsilon F,$$

where $\varepsilon = 1 - a/a_\infty$.

Attenuation of gravitational waves is due to the fact that at the frequencies which are of interest for experiment the tube radius variation is quasi-stationary whereas propagation of the wave requires that the kinetic and potential energies in it should be equal. To estimate the frequencies at which inertia manifests itself, we proceed from the equation for the radial component of the particle motion $m\ddot{a}=\delta F$. In view of (18), this equation is reduced to the form $\ddot{\varepsilon}=-2(c/a_\infty)^2(1+\varepsilon)^2\varepsilon$. In the linear approximation its solution is $\varepsilon=\varepsilon_0\cos(\omega t+\varphi)$, where ε_0 and φ are the vibration amplitude and phase, respectively, and $\omega=\sqrt{2}c/a_\infty=\sqrt{2}mc^2/\hbar$ is the angular frequency of vibrations many orders of magnitude higher than the frequency for receiving gravitational waves.

4. CONCLUSIONS

In this work simple physical substantiation is obtained on the basis of six-dimensional treatment of gravity for the Papapetrou metric (21): it is a consequence of applying the Fermat principle to trajectories of elementary particles moving in the complete space with the speed of light in compliance with the principle of simplicity. By applying the Einstein equation with two zero indices to metric (21), the coefficients of the metric are expressed in terms of elementary functions for the particular case of a spherically symmetrical static field. Both the gravitational acceleration from the point of view of the distant observer and the gravitational acceleration from the point of view of the local observer are everywhere finite and tend to zero as they approach the center of gravity. This means the absence of black holes.

Radii of real bodies in this metric should be larger than a quarter of their gravitational radii, otherwise the gravitational forces everywhere inside the body would be smaller than at its lower compression and thus the inner pressure would expand it. Therefore, outside the massive body this metric ensures monotonicity of the gravity force along the radial coordinate, so that there are no grounds for considering it as being unphysical. At the same time, radii of neutron stars are equal in order of magnitude to their gravitational radii. Therefore, the Papapetrou metric is not in conflict with existence of accretion discs in dense and rather massive stars.

In the six-dimensional treatment the gravitational force is a projection of the cosmological force, which confines particles to the Compton neighborhood of three-dimensional space, onto the meridian of the tubular envelope of the geodesic trajectories of elementary particles. Massive bodies decrease the speed of light in their neighborhood, and thus the motion tube radius decreases for the centrifugal force to be able to oppose the cosmological force. Consequently, the envelope takes the bottlelike form, which makes the meridian inclined with respect to the axis of the tube.

Gravity changes the radii of the motion tube and the speed of light on it, thus distorting metric spatial and temporal coefficients, but does not curve space itself from the point of view of the distant observer.

Gravitational waves are treated as perturbations over space and time of the radii of the motion tube and the speed of light on it. The equation for gravitational waves can also be obtained by applying the Einstein equation with two zero indices to metric (21). In this case the waves become inhomogeneous (analogous to inhomogeneous waveguide waves) and showing exponential attenuation with distance while moving farther and farther away from the source of perturbations. This does not allow direct reception of gravitational waves from celestial bodies even at distances like that to the Moon. However, it is possible to receive gravitational waves in the near field of a uniform rotator with n lobes, which is like the Maltese cross at $n=4$. It is preferable to try to detect gravitational waves at the harmonics of the rotator frequency.

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