Entanglement and causality in the interaction of the two-level atom with the field

2013 Phys. Scr. 88 055008

(http://iopscience.iop.org/1402-4896/88/5/055008)

IOP Publishing Physica Scripta
Phys. Scr. **88** (2013) 055008 (8pp) doi:10.1088/0031-8949/88/05/055008

Entanglement and causality in the interaction of the two-level atom with the field

E O Kiktenko[1,2] **and S M Korotaev**[1,2]

[1] Geoelectromagnetic Research Centre of Schmidt Institute of Physics of the Earth, Russian Academy of Sciences, PO 30 Troitsk, Moscow Region 142190, Russia
[2] Bauman Moscow State Technical University, 2nd Baumanskaya 5, 105005 Moscow, Russia

E-mail: evgeniy.kiktenko@gmail.com

Received 29 December 2012
Accepted for publication 23 September 2013
Published 23 October 2013
Online at stacks.iop.org/PhysScr/88/055008

Abstract
The interaction between an initially pure two-level atom and the mixed thermal quantized field mode in the framework of the Jaynes–Cummings model is considered with the employment of quantum causal analysis . At the high temperature of the field, a distinction between the resulting properties of initially excited and ground atom states smoothes over and the whole state turns out to be causally asymmetric, entangled and 'classical' in the entropic sense. The average asymptotic entanglement is found. It is revealed that the thermalization acting on the field corresponds to the stronger causality and entanglement decay, rather than the thermalization acting only on the atom.

PACS numbers: 03.65.Ud, 03.67.Mn

1. Introduction

Entanglement is an intriguing phenomenon in quantum systems, which has been intensively studied in recent years. On the one hand, it demonstrates surprising and important non-local features of our world and on the other hand, it appears to be a key resource for quantum computation, teleportation and other implementations of quantum information theory. There are some general issues in these implementations arising from the non-locality: the separation of quantum and classical correlations and the causality in the entangled states (because non-local correlations can be instantaneous and even advanced). The first issue has been clarified by quantum discord [1–3]. To clarify the second issue, a method of quantum causal analysis has been suggested recently. The latter gives a quantitative measure of causality based on information flow asymmetry regardless of the time relation. Besides a general understanding of the somewhat surprising features of quantum causality, the application of this method to various entangled states allowed physicists to gain insight into important practical phenomenon of the asymmetric fragility of these states to the different kinds of decoherence [4–7]. However, in these works, only the low-dimensional systems (two- and three-qubit mixed states) were considered. This paper is motivated by an interest in extending the causal analysis application to high-dimensional systems.

In this paper, we study the high-dimensional entanglement generated in the interaction between a two-level atom and the quantized resonant mode of a field, described by the commonly known Jaynes–Cummings model (JCM) [8]. It is a rather simple but effective model that has an analytical solution and which has been applied e.g. for describing the processes in the one-atom maser [9]. In [10], JCM was considered for the description of the interaction between a two-level atom initially in a pure state and a quantized resonant field mode initially in a mixed thermal state. Using the projection method on 2×2 systems, it was found that an entanglement created by the JCM interaction persists at the arbitrary high temperature of the field, but only the lower bound of it was estimated.

In this study, we do not use the projections and consider an analytical solution of corresponding von Neumann equation. Such treatment makes it possible to study the dynamics of any desired initial state. The problem of the infinite dimensionality of the field matrix is solved by taking into account the bounding of a number of fields' energy states. Such a treatment is quite appropriate because the thermal

distribution is exponentially decaying. Finite dimensionality makes it possible to use negativity as an entanglement measure.

Another interesting feature of the considered system is its asymmetry: an atom and a field have different entropic properties as a consequence of their different dimensionalities. Zyczkowski and Horodecki were the first to question the role of system asymmetry for quantum information transfer [11]. For a study of this asymmetry, we use quantum causal analysis. To observe the role of causality (information flow asymmetry), we consider a thermalization process which acts on the one subsystem (field or atom) together with the JCM interaction.

The paper is organized as follows. In section 2, the kernel of quantum causal analysis is presented. In section 3, we consider the JCM interaction and propose a model of thermalization process. Next we consider a step-wise evolution, which includes both the JCM interaction and the thermalization process. In section 4, we present computational results for different system characteristics during evolutions with and without the thermalization process and then discuss them in section 5. The conclusions are given in section 6.

2. Quantum causal analysis

Quantum causal analysis [4–7] is a new method which suggests the original approach to understanding of causality, previously developed at the classical level and widely applied to various classical physics problem (e.g. [12]). The essence of causal analysis is based on the formalization of the usual intuitive 'cause' and 'effect' concepts from the informational flow asymmetry of corresponding processes without invoking time relations. The retardation of an effect relative to the cause is introduced after their definition as an axiom.

To understand the principles of quantum causal analysis, let us consider a quantum bipartite state, which is characterized by the density matrix ρ_{AB} and consists of two subsystems A and B with reduced density matrices ρ_A and ρ_B respectively. Using these matrices, we can calculate corresponding von Neumann entropies S_A, S_B and S_{AB} by a general formula

$$S_X = -\mathrm{Tr}[\rho_X \log_2 \rho_X], \tag{1}$$

where X could be A, B or AB.

Mathematical formalization of causal analysis is founded on a pair of *independence functions*

$$\begin{aligned} i_{A|B} &= S_{A|B}/S_A, \\ i_{B|A} &= S_{B|A}/S_B, \end{aligned} \tag{2}$$

where $S_{A|B} = S_{AB} - S_B$ and $S_{B|A} = S_{AB} - S_A$ are conditional entropies. It is easy to see that for quantum variables $i \in [-1, 1]$, the independence functions are actually normalized conditional entropies. To understand the idea of independence functions, let us consider the main demonstrative cases. $i_{B|A} = -1$ (which can occur only when $i_{A|B} = -1$ too) means that the considered state is pure and entangled: $S_{AB} = 0$, $S_A = S_B \neq 0$, which corresponds to quantum correlations between two subsystems. If $i_{B|A} = 0$, then $S_{AB} = S_A$ and we obtain that the state B is one-valued function of the state A (notice that $i_{B|A} = 0$ does not imply $i_{A|B} = 0$). Therefore, in this context, we have maximal classical correlations. Further, in the case of $i_{B|A} = 1$, values of B are independent from A. Thus the smaller the value of $i_{B|A}$, the stronger the influence of A on B. It is very important that in the general case $i_{B|A} \neq i_{A|B}$, so independence functions characterizes *one-way correlations* between two subsystems in contrast to the mutual information

$$I_{AB} = S_A + S_B - S_{AB} = I_{AB}, \tag{3}$$

which characterizes total (quantum and classical) two-way correlations between the subsystems.

Another interesting feature of the independence functions is that they are non-negative ($i \in [0, 1]$) for classical and separable states (such states satisfy inequality $S_{AB} \geqslant \max(S_A, S_B)$ and their conditional entropies cannot be negative). In turn, negative values of the independence functions correspond to a non-classical phenomenon such as entanglement (e.g. as has already been mentioned for all pure entangled states $i_{A|B} = i_{B|A} = -1$). Therefore, the independence functions can indicate whether the system is 'quantum' or 'classical' in the entropic sense. If at least one independence function is negative ($i_{A|B} < 0$ or $i_{B|A} < 0$), then the system should be called 'quantum'. If both $i_{A|B} \geqslant 0$ and $i_{B|A} \geqslant 0$, then the system should be called 'classical'. It is worth noting that similar definitions were proposed in [11] (there the authors considered 'quantum-classical' bipartite state AB, where the A subsystem was 'quantum' with $i_{B|A} < 0$ and B was 'classical' with $i_{A|B} > 0$). In [4, 5], the authors uncovered that it is possible to have a situation when a mixed state has both positive independence functions but is still entangled. So separability is an *efficient but not necessary* condition for the non-negativity of independence functions. Below we will show that such a situation is quite typical for an atom–field interaction.

There is an obvious qualitative agreement between the independence functions and quantum discord [1–3], which is defined as

$$D_{AB} = I_{AB} - \max_{\{\Pi_k^B\}} J_{AB}^{\{\Pi_k^B\}}, \tag{4}$$

where $\{\Pi_k^B\}$ is the complete set of positive operator valued measurements of B and $J_{AB}^{\{\Pi_k^B\}}$. This is the mutual information of the state $\widetilde{\rho}_{AB}^{\{\Pi_k^B\}} = \sum_k \Pi_k^B \rho_{AB} \Pi_k^B$, where the original state ρ_{AB} after the measurement is $\{\Pi_k^B\}$. Quantum discord measures the maximum amount of locally inaccessible information and it is widely used as a measure of the quantumness of correlations. The greater the discord is, the greater the quantum correlations are.

In line with (4) we can obtain an expression for D_{BA}, which corresponds to measurement of A. There are mixed states with asymmetrical discords $D_{AB} \neq D_{BA}$. Compared to the independence functions, the quantum discord computation is more complicated because it involves projection measurements with a search of the optimal basis. Also there are property distinctions, which are more essential. The quantum discord can be non-zero for separable states, while both the independence functions *cannot* be negative for these states. However, the main point is that a derivation of the causality measure via the standard theorem about the information flow rate through a noisy channel is possible

only in terms of the independence function pair [4, 5]. Thus, although some physical results gained from the quantum discord and independence functions may be similar, the main problem of causal analysis is resolved precisely in terms of the independence functions.

In the causal analysis, the independence functions are used to determine a direction and a strength of causality, accordingly to inequality $i_{B|A} \neq i_{A|B}$. To measure the causal connection between subsystems A and B, we use c_2^{AB}, called the course of time (notation follows Kozyrev's pioneering work on causal mechanics [13]), derived in [4, 5] as the velocity of an irreversible information flow:

$$c_2^{AB} = k\frac{(1-i_{A|B})(1-i_{B|A})}{i_{A|B}-i_{B|A}}, \tag{5}$$

where $k = \Delta r/\delta t$. Δr is the effective distance between A and B, while δt is the time of the brachistochrone evolution [14]. For orthogonal states

$$\delta t = \frac{\pi\hbar}{2(\Delta E)_{\max}}, \tag{6}$$

where $(\Delta E)_{\max}$ is the difference between the largest and the smallest eigenvalues of the Hamiltonian.

The sign of c_2^{AB} is specified by the direction of the causal connection: $c_2^{AB} > 0$ ($i_{A|B} > i_{B|A}$) means that subsystem A is the 'cause' (informational source) and B is the 'effect' (informational sink), while $c_2^{AB} < 0$ ($i_{A|B} < i_{B|A}$) means that B is the 'cause' and A is the 'effect' ($c_2^{AB} = -c_2^{BA}$). The strength of the causal connection corresponds to the absolute value $|c_2^{AB}|$: the stronger the causality is, the less $|c_2^{AB}|$ is (in the case of a constant value of total correlations, defined by the mutual information (3)). It is noteworthy that e.g. for all pure entangled states $|c_2^{AB}| \to \infty$, this totally conforms with the conventional view of quantum correlations as causeless and instantaneous. However, in mixed states independence functions do not need to be equal, therefore causality may occur.

As it was shown in [4], the coefficient k qualitatively does not influence the course of time, so hereafter we set $k = 1$ and deal only with the dimensionless factor of (5).

Cramer [15] was the first to distinguish the principles of strong and weak causality. The strong causality corresponds to the usual condition of retardation $\tau_{A\to B}$ of the effect relative to the cause

$$\begin{aligned} c_2^{AB} > 0 &\Rightarrow \tau_{A\to B} > 0, \\ c_2^{AB} < 0 &\Rightarrow \tau_{A\to B} < 0, \\ |c_2^{AB}| \to \infty &\Rightarrow \tau_{A\to B} \to 0. \end{aligned} \tag{7}$$

Without the condition (7) we have weak causality, which corresponds only to non-local correlations. Even as they occur in reverse time, they only relate to the unknown states (hence the 'telegraph to the past' is impossible). Although it is not very important for the scope of our work, note that weak causality admits the extraction of information from the future without well known classical paradoxes. The experimental possibility of the detection of such a time reversal phenomena was theoretically predicted by Elitzur and Dolev [16] and has really been proved for quantum teleportation [17, 18], entanglement swapping [19, 20] and macroscopic entanglement, e.g. [21]. Note that we do not use the axiom (7) anywhere in this paper.

The main result of previous quantum causal analysis implementation consists in prediction of the degree of entanglement fragility with respect to the different decoherence processes of two- [4, 6] and three- [5, 7] qubit systems. In this paper, we consider, for the first time, a high-dimensional system with an interesting feature: it contains an asymmetry which is based on a difference in the physics of the objects. It makes the implementation of quantum causal analysis a promising area for the study of entanglement creation by JCM interaction.

3. Model of interaction

3.1. Jaynes–Cummings Hamiltonian

We consider the bipartite system, which consists of the two-level atom, that can be found in the ground state $|g\rangle_a$ and the excited state $|e\rangle_a$. We also consider the quantized mode of the field with possible energy states $|0\rangle_f$, $|1\rangle_f$, $|2\rangle_f$, ... For simplification, we set the detuning frequency to zero (resonance case is considered).

For this system, the JMC Hamiltonian is

$$H = \tfrac{1}{2}\hbar\omega\sigma_z + \hbar\omega a_f^\dagger a_f + \hbar g(|e\rangle\langle g|_a a_f + |g\rangle\langle e|_a a_f^\dagger), \tag{8}$$

where σ_z is the z-component of the Pauli matrix, ω is the resonance frequency, $a_f^\dagger$ and a_f are the creation and annihilation operators of the field mode respectively and g is the dipole matrix element, which defines the Rabi frequency. It is helpful to write the Hamiltonian of the full system as a sum of two commuting parts: $H = H_0 + V$, where $H_0 = \frac{1}{2}\hbar\omega\sigma_z + \hbar\omega a_f^\dagger a_f$ is a diagonal matrix and $V = \hbar g(|e\rangle\langle g|_a a_f + |g\rangle\langle e|_a a_f^\dagger)$ is a matrix with only off-diagonal elements, that corresponds to the interaction the between subsystems. The dynamics of the system is described by the von Neumann equation

$$\mathrm{i}\hbar\frac{\partial\rho_{\mathrm{af}}(t)}{\partial t} = [H, \rho_{\mathrm{af}}(t)], \tag{9}$$

where $\rho_{\mathrm{af}}(t)$ is the density matrix of the whole system and the brackets denote a commutator: $[A, B] \equiv AB - BA$. The Hamiltonian (8) is time independent, so the solution of (9) takes the form

$$\rho_{\mathrm{af}}(t) = \mathrm{e}^{-\mathrm{i}Ht/\hbar}\rho_{\mathrm{af}}^0\,\mathrm{e}^{\mathrm{i}Ht/\hbar}, \tag{10}$$

where $\rho_{\mathrm{af}}^0 = \rho_{\mathrm{af}}(0)$ is the density matrix of the initial state at $t = 0$.

In all our subsequent calculations, we set $\hbar$ and g to unity. For the resonance case, we have $[H_0, V] = 0$; so when ρ_{af}^0 is diagonal, the solution (10) is defined only by the interaction part V

$$\rho_{\mathrm{af}}(t) = \mathrm{e}^{-\mathrm{i}Vt}\rho_{\mathrm{af}}^0\,\mathrm{e}^{\mathrm{i}Vt}. \tag{11}$$

3.2. Thermalization process

Thermalization is a process in which a system reaches a state of thermal equilibrium due to the interaction with a bath. Let us consider the thermal states of the field and atom.

In line with [10], the mixed thermal state for the field is of the form

$$\rho_f^{\mathrm{T}} = \sum_{i=0}^{\infty} P_i |i\rangle\langle i|_f, \tag{12}$$

where P_i are the probability distribution coefficients. As the field satisfies Bose–Einstein statistics, we have

$$P_i = \frac{1}{1+\langle n\rangle}\left(\frac{\langle n\rangle}{1+\langle n\rangle}\right)^{\mathrm{i}} \tag{13}$$

with mean photon number

$$\langle n\rangle = \frac{1}{\mathrm{e}^{\hbar\omega/k_{\mathrm{B}}T}-1}, \tag{14}$$

where k_{B} is the Boltzmann constant and T is the temperature.

For the atom, the thermal state takes the form $\rho_a^{\mathrm{T}} = \lambda|g\rangle\langle g|_a + (1-\lambda)|e\rangle\langle e|_a$, where $(1-\lambda)/\lambda = \exp(-\Delta E/k_{\mathrm{B}}T)$ (ΔE being the energy difference between $|e\rangle$ and $|g\rangle$). For the resonance case $\Delta E = \hbar\omega$ and using (14), we can easily obtain that

$$\lambda = \frac{1+1/\langle n\rangle}{2+1/\langle n\rangle}. \tag{15}$$

In line with [22, 23], we can introduce a model of the thermalization process for an *each* subsystem as a transformation of the corresponding matrix elements. For the field thermalization, this transformation Λ_f takes the form

$$\begin{aligned} |n\rangle\langle n|_f &\to (1-p_f)|n\rangle\langle n|_f + p_f\sum_{i=0}^{\infty}[P_i|i\rangle\langle i|_f], \\ |n\rangle\langle m|_f &\to (1-p_f)|n\rangle\langle m|_f \text{ for } n\neq m, \end{aligned} \tag{16}$$

where $n, m = 0, 1, 2, \ldots$ and $p_f \in [0, 1]$ define the field thermalization degree and thermal distribution coefficients P_i could be calculated from (13).

Similarly, the thermalization process of the atom Λ_a can be described by the following transformation:

$$\begin{aligned} |g\rangle\langle g|_a &\to (1-p_a)|g\rangle\langle g|_a + p_a[\lambda|g\rangle\langle g|_a + (1-\lambda)|e\rangle\langle e|_a], \\ |e\rangle\langle e|_a &\to (1-p_a)|e\rangle\langle e|_a + p_a[\lambda|g\rangle\langle g|_a + (1-\lambda)|e\rangle\langle e|_a], \\ |g\rangle\langle e|_a &\to (1-p_a)|g\rangle\langle e|_a, \\ |e\rangle\langle g|_a &\to (1-p_a)|e\rangle\langle g|_a, \end{aligned} \tag{17}$$

where $p_a \in [0, 1]$ defines the atom thermalization degree and the thermal coefficient λ can be calculated from (15).

As we see, these transformations turn the corresponding subsystem into a mixed thermal state and also destroy correlations with another subsystem.

It should be noted that the action of the thermalization on both the field and the atom turns the system into a separable state $\rho_a^{\mathrm{T}} \otimes \rho_f^{\mathrm{T}}$, which is approximately equal (as $\omega \gg g$ in (8)) to usual thermal state $\rho_{\mathrm{af}}^{\mathrm{T}} = \exp(H/k_{\mathrm{B}}T)/\mathrm{Tr}[\exp(H/k_{\mathrm{B}}T)]$.

3.3. Time evolution model

We consider the case in which the atom and field are involved in two processes: the first is the interaction with each other by JCM and the second is the interaction of one subsystem (atom or field) with the environment (bath) by thermalization.

In line with [11], let us consider the following step-wise evolution:

$$\begin{aligned} \rho_{\mathrm{af}}^0 \to \rho_{\mathrm{af}}^{0*} = \mathrm{e}^{-\mathrm{i}V\Delta t}\rho_{\mathrm{af}}^0\,\mathrm{e}^{\mathrm{i}V\Delta t} \to \rho_{\mathrm{af}}^1 = \Lambda_x[\rho_{\mathrm{af}}^{0*}] \to \\ \to \rho_{\mathrm{af}}^{1*} = \mathrm{e}^{-\mathrm{i}V\Delta t}\rho_{\mathrm{af}}^1\,\mathrm{e}^{\mathrm{i}V\Delta t} \to \ldots, \end{aligned} \tag{18}$$

where Λ_x corresponds either to the transformation Λ_f (16) or to the transformation Λ_a (17). As we see, the evolution from ρ_{af}^i to ρ_{af}^{i*} corresponds to the interaction by JMC during a small period Δt in line with solution (11). The transformation from ρ_{af}^{i*} to ρ_{af}^{i+1} corresponds to the weak instantaneous interaction between one of the subsystems with an environment. Note that we should set the thermalization degree in Λ_x to be rather small ($p_x \ll 1$).

As a result, we can sequentially obtain a set of $\{\rho_{\mathrm{af}}^n\}$, which corresponds to the states of the atom–field system at $t = n\Delta t$. Then we can obtain the evolution of causality and entanglement, which is the main object of study in the rest of this paper.

It is worth noting that, in practice, the thermalization process acts on both subsystems simultaneously. In our consideration, we suppose that thermalization acts on one subsystem much stronger than on another subsystem: e.g. in the case of only field thermalization, we consider $p_f \gg p_a$ so we can neglect Λ_a transformation. Such an novel model helps us to understand the role of causality in the evidently asymmetric system.

4. Computation results

4.1. Evolution without thermalization

First let us consider the evolution of the system without an influence of thermalization, which corresponds to the solution (11) or the step-wise evolution (18) with $p_{f(a)} = 0$ in the transformation $\Lambda_{f(a)}$.

In [10], a JCM evolution was considered of the initial state $\rho_{\mathrm{af}}^{0e} = |e\rangle\langle e|_a \otimes \rho_f^{\mathrm{T}}$. It has been found that the initial purity of the atom involves an appearance of entanglement in such a mixed system, but only the lower bound of this entanglement has been obtained.

The main computational problem for that system is the infinite dimensionality of the field matrix ρ_f. For the thermal distribution (or very close to it), this problem could be solved by a confinement of the field matrix. Indeed, it is evident from equation (13) that P_i is an exponentially decaying series so that the contribution of the matrix elements $P_i|i\rangle\langle i|_f$ at sufficiently high i vanishes. Therefore, in our calculations, we can confine series P_i at $i = N_{\max} - 1$ and estimate the occurred error as $\alpha = \frac{\langle n\rangle - \sum_{n=0}^{N_{\max}-1}[nP_n]}{\langle n\rangle}$ (this is an energy amount which we have lost by the confinement). For our computations, we chose $N_{\max}$ so that $\alpha < 1\%$.

Using the density matrix $\rho_{\mathrm{af}}(t)$ (11), we can compute the reduced atom and field density matrices: $\rho_a(t) = \mathrm{Tr}_f\rho_{\mathrm{af}}(t)$ and $\rho_f(t) = \mathrm{Tr}_a\rho_{\mathrm{af}}(t)$, where Tr_f and Tr_a denote partial traces

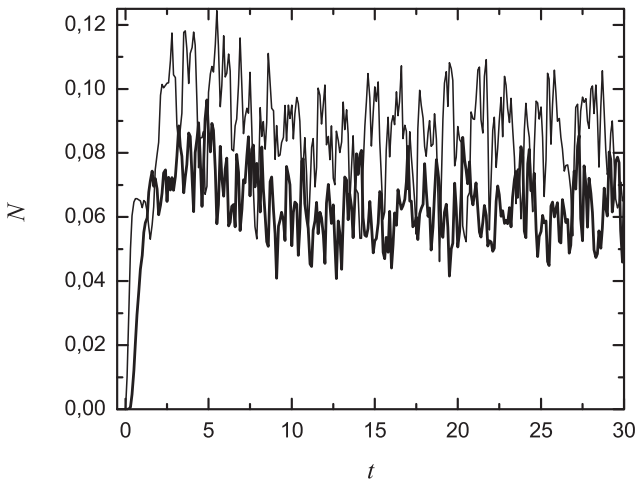

Figure 1. Negativity of $\rho^{e}_{\mathrm{af}}(t)$ (thin line) and $\rho^{g}_{\mathrm{af}}(t)$ (bold line) at $\langle n\rangle = 10$.

with respect to the corresponding subsystems. From these three matrices we can obtain von Neumann entropies of the whole system $S_{\mathrm{af}}(t)$ and two subsystems $S_a(t)$ and $S_f(t)$ by equation (1). Then we can compute mutual information (3) and the independence functions (2), which determine the course of time (5). As we shall see further, generally $i_{f|a}(t) > i_{a|f}(t)$, so we consider $c_2^{af}(t)$ to deal with the positive values.

We use the negativity as a standard measure of bipartite entanglement, which is defined as (e.g. [24])

$$N[\rho_{\mathrm{af}}] = \sum_i |\mu_i|, \tag{19}$$

where μ_i are the negative eigenvalues of $\rho^{\mathrm{T}_a}_{\mathrm{af}}$ and T_a denotes the partial transpose with respect to the first atom. According to equation (19), the maximal entanglement corresponds to $N = 0.5$.

We consider two initial states: $\rho^{0e}_{\mathrm{af}} = |e\rangle\langle e|_a \otimes \rho^{\mathrm{T}}_f$ and $\rho^{0g}_{\mathrm{af}} = |g\rangle\langle g|_a \otimes \rho^{\mathrm{T}}_f$ where the atom is either in an excited or a ground pure state and the field is always in the thermal mixed state. Their dynamics are denoted as $\rho^{e}_{\mathrm{af}}(t)$ and $\rho^{g}_{\mathrm{af}}(t)$, respectively.

In figure 1, the negativity dynamics of $\rho^{e}_{\mathrm{af}}(t)$ and $\rho^{g}_{\mathrm{af}}(t)$ at $\langle n\rangle = 10$ are shown. As it was predicted in [10] that entanglement appears at the beginning of an interaction and then never drops to zero. It is notable that there is some average value of negativity, which is retained during the whole evolution process (except for the very short period at the beginning).

Other features of the system, such as the mutual information I and the causality c_2^{fa}, also have analogous dynamics; so it seems logical to estimate their average values as functions of the mean photon number $\langle n\rangle$. We have chosen time series $150 \leqslant t \leqslant 400$ with the time step $\mathrm{d}t = 0.5$ and have computed the average values $N_{(\mathrm{av})}$, $I_{(\mathrm{av})}$, $i_{f|a(\mathrm{av})}$, $i_{a|f(\mathrm{av})}$, and $c^{fa}_{2(\mathrm{av})}$ of the negativity, mutual information, both the independence functions and the course of time, respectively, for the set of mean photon numbers $1 \leqslant \langle n\rangle \leqslant 80$ and for the same initial states (ρ^{0e}_{af} and ρ^{0g}_{af}). The $t_{\min} = 150$ has been chosen to avoid getting in a period of transfer to the quasi-stationary state. The time step $\mathrm{d}t = 0.5$ has been chosen as it does not correspond to any system eigenfrequencies. Together with the average values, we have stored the minimal and maximal values of the characteristics at $\langle n\rangle \geqslant 10$ to observe the corresponding sizes of deviations from averages at rather high temperatures.

The results of such averaging are presented in figure 2. The general feature for all parameters is that the higher the $\langle n\rangle$ is, the closer the average values for two different initial states are. Moreover, the extent of the fluctuation for all the parameters decreases with the temperature growth.

Figure 2(a) demonstrates the dependence of negativity $N_{(\mathrm{av})}$ on $\langle n\rangle$. It is expectable that for an initially pure excited state of the atom, entanglement decreases with the temperature rise, but it surprisingly does not vanish. It tends to an asymptotic value, as well as the curve for the initial ground state. It is remarkable that for the initial ground atom state. there is an amplification of entanglement with the growth of the temperature, so in this case the temperature plays a constructive role for the entanglement generation.

Figure 2(b), which corresponds to the dynamics of information $I_{(\mathrm{av})}$, demonstrates features similar to figure 2(a). This was quite expected, as both the entanglement and mutual information are measures of the correlations in the system.

Figure 2(c) demonstrates that $i_{f|a(\mathrm{av})}$ is always larger than $i_{a|f(\mathrm{av})}$, so the correlations between the atom and field are asymmetric and the field always corresponds to the cause (source of information) while the atom always corresponds to the effect (sink of information). It is very intriguing that at rather high temperatures, both the $i_{f|a}$ and $i_{a|f}$ are always positive, while the negativity N is also greater than zero. It means that the considered system turns out to be 'classical' in the entropic sense but is still entangled.

Finally, figure 2(d) shows that for both the initial states, causality amplifies with $\langle n\rangle$ growth. It is quite expected for the initial state ρ^{g0}_{af} as irreversible information flow can be associated with the energy flow running from the field to the atom. But for the state ρ^{e0}_{af}, this result is non-trivial and does not conform with intuition. Another interesting result is that for ρ^{g0}_{af}, there is an amplification of both the causality and the entanglement as the temperature increases. Indeed, causality needs mixedness, which is usually harmful for entanglement, but in this situation mixedness of the field turns out to be a necessary condition for the entanglement appearance.

From figure 2 we can also estimate some asymptotic values of the correlations at $\langle n\rangle \gg 0$ for both the initial states: $N_{(\mathrm{as})} \simeq 0.07$ (which is 14% of the maximum value), $I_{(\mathrm{as})} = 0.8$ bit (which is 40% of the maximum value). The system turns out to be entangled, classical in the entropic sense and information-wise is asymmetric: the field state is the cause with respect to the atom state.

4.2. Evolution with thermalization

Now let us consider a situation when one of the two subsystems is influenced by an environment with the thermalization. Take the two states: $\rho^{\mathrm{Tf}}_{\mathrm{af}}(t)$ and $\rho^{\mathrm{Ta}}_{\mathrm{af}}(t)$, which have been obtained from the step-wise evolution (18) with $\Lambda_x = \Lambda_f$ and $\Lambda_x = \Lambda_a$ respectively. For the initial state, we have chosen $\rho^{0}_{\mathrm{af}} = \rho^{e0}_{\mathrm{af}}$ with $\langle n\rangle = 1$. This state is quite interesting because it is *energy-wise symmetric* but, as we

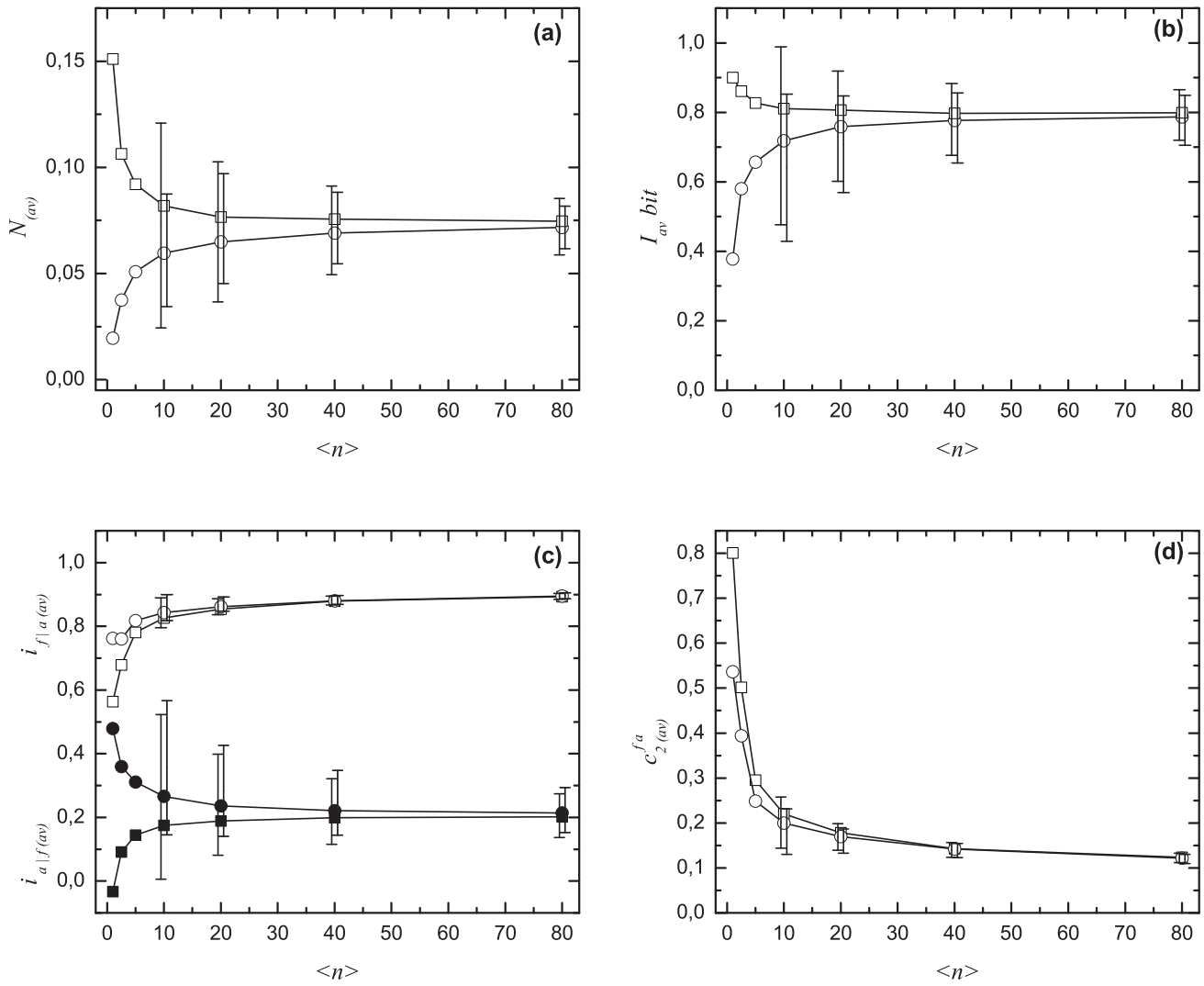

Figure 2. Time averaged characteristics and corresponding minimal and maximal values (only for $\langle n\rangle \geqslant 10$) as functions of $\langle n\rangle$ for different initial states: $\rho_{\rm af}^{e0}$ (squares and right vertical lines) and $\rho_{\rm af}^{g0}$ (circles and left vertical lines). Averaged characteristics: (a) negativity $N_{\rm (av)}$; (b) mutual information $I_{\rm (av)}$; (c) independence functions $i_{f|a{\rm (av)}}$ (empty symbols) and $i_{a|f{\rm (av)}}$ (filled symbols); and (d) causality $c^{fa}_{2{\rm (av)}}$.

have seen before, interaction by JCM induces a causality (information-wise asymmetry).

The results for $\Delta t = 0.1$ at $p_f = 0.01$ and $p_a = 0.01$ are presented in figure 3. From figure 3(a) we see that the entanglement in both cases is decreasing (as is quite expected) but in case of the field thermalization it decays generally more intensively than in the case of atom thermalization. The causality dynamics, which is shown in figure 3(b), demonstrates an amplification of the causal connection. Also, there is a simple relationship between entanglement and causality: the entanglement is weaker in the state with stronger causality.

5. Discussion

The first result which has been obtained in section 4.1 is that the average level of entanglement created by the JCM interaction between an initially pure atom and mixed thermal field at high temperatures does not depend on temperature and has a rather high level ($\simeq$14% of maximal value for negativity). So e.g. if we have an atom–field system in totally thermal separable state $\rho_a^{\rm T} \otimes \rho_f^{\rm T}$ and then measure the atom state—we obtain a system in state $|e\rangle\langle e|_a \otimes \rho_f^{\rm T}$ or $|g\rangle\langle g|_a \otimes \rho_f^{\rm T}$. If we isolate both subsystems from the environment and let them interact with each other by the JCM model, after some period they become entangled. Finally we may say that in the considered case, *the measurement is a source of purity and causes high-temperature entanglement*. Such a source of entanglement can likely be used in different applications.

The second result corresponds to causal analysis implementation. As it is shown in [4, 5], mixedness is a necessary condition for quantum causality appearance. In [4–7] different situations have been considered where decoherence induces causality. In section 4.1, we considered the case where causality appears in the state, which had already been mixed at the stage of state preparation, when the system was open. Then we isolated the system from the environment and let it evolve in accordance with the time-independent JCM Hamiltonian. During this evolution, the mixedness of the whole state kept its value but the causality changed in time, although it retained a definite direction: the field was the cause (source of information)

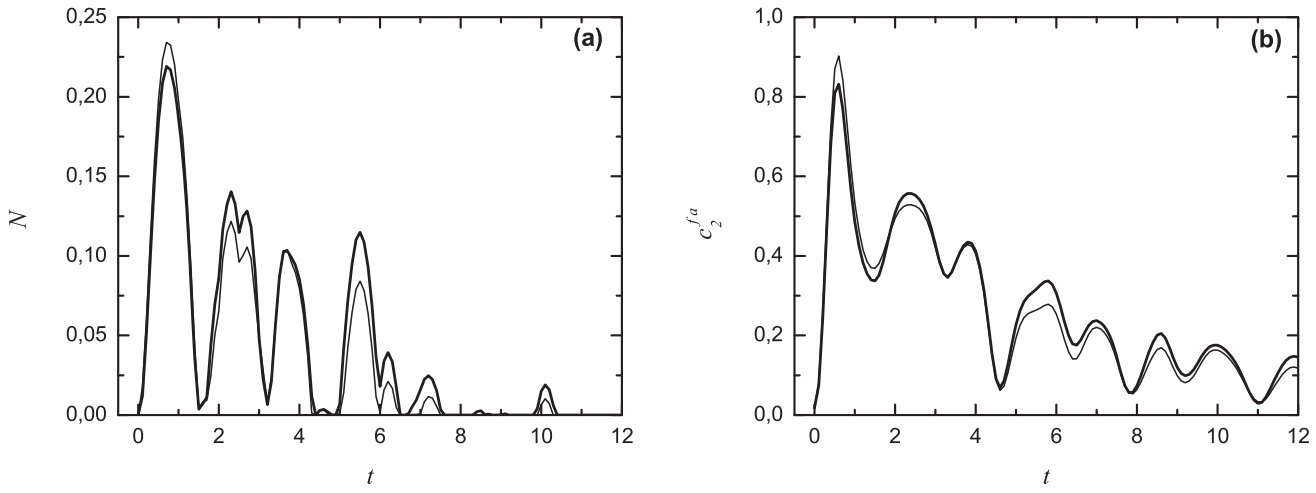

Figure 3. Negativity N (a) and causality c_2^{fa} (b) of states $\rho_{\mathrm{af}}^{\mathrm{Tf}}(t)$ (thin lines) and $\rho_{\mathrm{af}}^{\mathrm{Ta}}(t)$ (bold lines).

and the atom was the effect (sink of information). With a rise of the initial temperature of the field, the average strength of the causality amplified. It totally corresponds to the fact that the field is an infinite dimensional subsystem and is able to keep more information than the two-level atom. Also it is remarkable that the entanglement created by the JCM interaction corresponds to 'classical' correlations in the entropic sense. Such a peculiarity is typical for macroscopic entanglement observations [21].

To explain such a situation in section 4.2 we should realize what the thermalization process is. It can be considered touching the global source of information (which the bath is). After this touching, the corresponding subsystem not only loses correlation with another subsystem (which does not connected with the bath) but also tends to be a cause for it. For our system, the process of correlation destruction always corresponds to $|c_2^{fa}| \to 0$. However, in the case of the atom thermalization, the environment tries to inverse the existing causal connection from the field to the atom, so in this situation c_2^{fa} generally decreases slowly, rather than in the case of the field thermalization when this thermalization induces causality in the same direction as the JCM evolution and amplifies it. As an asymmetry is harmful for the entanglement, it is quite expected that negativity decays more intensively in the case of the field thermalization.

6. Conclusion

We have considered the high dimensional system which consists of the two-level atom and infinite-level resonance quantized mode of the field. Dynamics of such a system is described by JCM with Hamiltonian (8). We have studied different characteristics of an analytical solution (11) of corresponding von Neuman equation (9) for the two different initial states. In agreement with Bose *et al* [10], we have obtained that a JCM interaction between the atom in a pure state with the field in a thermal mixed state generates entanglement. As the initial field temperature increases, the average value of entanglement tends to some asymptotic value ($\simeq 14\%$ of the maximal value as measured by negativity). In the case of an atom in the initially ground pure state, the average entanglement grows as the initial temperature of the field increases. As purity of the initial atom state can be obtained by its measurement, the latter could serve as a source of high-temperature entanglement.

The above conclusion seems to not be very surprising and might have been obtained before, while the below ones are non-trivially new.

The key result is that the considered states are information-wise asymmetric: the field is the cause and the atom is the effect under any conditions, while the strength of the causal connection amplifies as the field initial temperature increases. It is interesting that states turn out to be 'classical' in the entropic sense in spite of an entanglement presence. Adding a thermalization process to one of the subsystems to the JCM evolution results in entanglement and information decay with an amplification of the causality. In the case of atom thermalization, entanglement decreases not so intensively as in case of the field thermalization, which can be explained by the opposite influence on causality. The JCM evolution with the atom thermalization induces a lesser asymmetry than the JCM evolution with the field asymmetry.

Finally it should be mentioned that the last result seems to be opposite to the results of [6, 7], where the decoherence of an *initially asymmetric two-qubit entangled state* (with finite causality strength) was considered. A decoherence process acting against the original causality destroys entanglement to a greater extent than such a process acting along this causality. The fact is that in this paper, we have considered the process which creates causality and entanglement in an *initially separable asymmetric state* and have uncovered a destructive role of the information-wise asymmetry for entanglement.

Acknowledgments

This work was supported by RFBR (grant numbers 12-05-00001 and 12-05-98009) and the Council for Grants of the President of the Russian Federation (grant number SP-961.2013.5).

References

[1] Henderson Vedral V 2001 *J. Phys. A: Math. Gen.* **34** 6899
[2] Olliver H and Zurek W H 2002 *Phys. Rev. Lett.* **88** 017901

[3] Fanchini F F, Castelano L K, Cornelio M F and de Oliveira M S 2012 *New J. Phys.* **14** 013027
[4] Korotaev S M and Kiktenko E O 2010 *AIP Proc.* **1316** 295
[5] Korotaev S M and Kiktenko E O 2011 *Physical Interpretation of Relativity Theory* ed P Rowlands (Moscow: BMSTU PH) p 201
[6] Kiktenko E O and Korotaev S M 2012 *Phys. Lett.* A **376** 820
[7] Korotaev S M and Kiktenko E O 2012 *Phys. Scr.* **85** 055006
[8] Jaynes E T and Cummings F W 1963 *Proc. IEEE* **51** 89
[9] Walther H 1996 *Phys.—Usp.* **39** 727
[10] Bose S, Fuentes-Guridi I, Knight P L and Vedral V 2001 *Phys. Rev. Let.* **87** 050401
[11] Zyczkowski K, Horodecki P, Horodecki M and Horodecki R 2001 *Phys. Rev.* A **65** 012101
[12] Korotaev S M 1992 *Geomagn. Aeronomy* **32** 27
[13] Kozyrev N A 1971 *Time in Science and Philosophy* ed J Zeman (Prague: Academia) p 111
[14] Borras A, Plastino A R, Casas M and Plastino A 2008 *Phys. Rev.* A **78** 052104
[15] Cramer J G 1980 *Phys. Rev* D **22** 362
[16] Elitzur A S and Dolev S 2003 *The Nature of Time: Geometry, Physics and Perception* ed R Buccery, M Saniga and W M Stuckey (Dordrecht: Kluwer) p 297
[17] Laforest M, Baugh J and Laflamme R 2006 *Phys. Rev.* A **73** 032323
[18] Lloyd S *et al* 2011 *Phys. Rev. Lett.* **106** 040403
[19] Ma X-S *et al* 2012 *Nature Phys.* **8** 480
[20] Megidish E *et al* 2013 *Phys. Rev. Lett.* **110** 21040
[21] Korotaev S M and Serdyuk V O 2008 *Int. J. Comput. Anticipatory Syst.* **20** 31
[22] Jang S S, Cheong Y W, Kim J and Lee H W 2006 *Phys. Rev.* A **74** 062112
[23] Song W and Chen Z-B 2007 *Phys. Rev.* A **76** 014307
[24] Wang X and Wang Z D 2006 *Phys. Rev.* A **73** 064302