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ELEMENTARY PARTICLES AND FIELDS
Theory

Algebrodynamics Over Complex Space and Phase Extension of the Minkowski Geometry*

V. V. Kassandrov**

Institute of Gravitation and Cosmology, Peoples' Friendship University, Moscow, Russia

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Abstract—First principles should predetermine physical geometry and dynamics both together. In the "algebrodynamics" they follow solely from the properties of biquaternion algebra $\mathbb{B}$ and the analysis over $\mathbb{B}$. We briefly present the algebrodynamics over Minkowski background based on a nonlinear generalization to $\mathbb{B}$ of the Cauchi—Riemann analyticity conditions. Further, we consider the effective real geometry uniquely resulting from the structure of $\mathbb{B}$ multiplication and found it to be of the Minkowski type, with an additional phase invariant. Then we pass to study the primordial dynamics that takes place in the complex $\mathbb{B}$ space and brings into consideration a number of remarkable structures: an ensemble of identical correlated matter pre-elements ("duplicons"), caustic-like signals (interaction carriers), a concept of random complex time resulting in irreversibility of physical time at macrolevel, etc. In partucular, the concept of "dimerous electron" naturally arises in the framework of complex algebrodynamics and, together with the above-mentioned phase invariant, allows for a novel approach to explanation of quantum interference phenomena alternative to recently accepted wave—particle dualism paradigm.

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1. STATUS OF MINKOWSKI GEOMETRY AND THE ALGEBRODYNAMICAL PARADIGM

A whole century after German Minkowski introduced his famous conception of the 4D space—time continuum, we come to realize the restricted nature of this conception and the necessity of its revision, supplement and derivation from some general and fundamental principle.

Indeed, formalism of the 4D space—time geometry was indispensable to ultimately formulate the Special Theory of Relativity (STR), to ascertain basic symmetries of fundamental physical equations and related conservation laws. It was also the Minkowski geometry that served as a base for formulation of the concept of *curved* space—time in the framework of the Einstein's General Theory of Relativity (GTR).

Subsequently, Minkowski geometry and its pseudo-Riemannian analog have been generalized via introduction of *effective* geometries related to correspondent field dynamics (in the formalism of *fiber bundles*), or via exchange of Riemannnian manifold for spaces with torsion, nonmetricity or additional "hidden" dimensions (in the *Kaluza—Klein formalism*). There have been considered also the models of discrete space—time, the challenging scheme of *causal sets* [1] among them.

However, none of modified space—time geometries has become generally accepted and able to replace the Minkowski geometry. Indeed, especial significance and reliability of the latter is stipulated by its origination from trustworthy physical principles of STR and, particularly, from the structure of experimentally verificated Maxwell equations. None of its subsequent modifications can boast of such a firm and uniquely interpreted experimental ground.

From the epoch of Minkowski we did not get better comprehension of the true geometry of our World, its hidden structure and origination. In fact, we are not even aware whether physical geometry is Riemannian or flat, has four dimensions or more, etc. Essentially, we can say nothing definite about the *topology* of space (both global and at microscale). And, of course, we still have no satisfactory answer to sacramental question: "Why is the space three dimensional (at least, at macrolevel)?" Finally, an "eternal" question about the sense and origin of *physical time* stands as before on the agenda.

Meanwhile, the Minkowski geometry suffers itself from grave shortcomings, both from phenomenological and generic viewpoints. To be concrete, complex

*The text was submitted by the author in English.

**E-mail: vkassan@sci.pfu.edu.ru

structure of field equations accepted in quantum theory results, generally, in string-like structure of field singularities (perhaps, it was first noticed by Dirac [2]) and, moreover, these strings are unstable and, as a rule, radiate themselves to infinity (see, e.g., [3] and the example in Section 2).

Another drawback (exactly, insufficiency) of the Minkowski geometry is the absence of fundamental distinction of temporal and spatial coordinates within its framework. Time enters the Minkowski metrical form on an equal footing with ordinary coordinates though with opposite sign. In other words, in the framework of the STR geometry time does not reveal itself as an *evolution parameter* as it was even in the antecedent Newton's picture of the World. At a pragmatic level this results, in particular, in the difficulty to coordinate "times" of various interacting (entangled) particles in an ensemble, in impossibility to introduce universal *global* time and to adjust the latter to *proper* times of different observers, or in the absence of clear comprehension of the passage of *local* time and dependence of its rate on matter. All these problems are widely discussed in physical literature (see, e.g., [4]) but are still far from resolution.

However, the main discontent with generally accepted Minkowski geometry is related to the fact that this geometry does not follow from some deep *logical* premises or exceptional *numerical* structures. This is still more valid with respect to generalizations of space–time structure arising, in particular, in the superstring theories (11D spaces) and in other approaches for purely phenomenological, "technical" reasons which in no way can replace the transparent and general physical principles of STR, of relativity and of universal velocity of interaction propagation.

At present, physics and mathematics are mature enough for construction of multidimensional geometries with different number of spatial and temporal dimensions. Moreover, they aim to create a general unified conception from which it would follow definite conclusions on the *true* geometry of physical space and on the properties and meaning of physical time, on the *dynamics of Time* itself!

In most of approaches of such kind the Minkowski space does not reproduce itself in its canonical form but is either deformed through some parameter (say, fundamental length and mass in the paradigm of Kadyshevsky [5]) under correspondence with canonical scheme, or changes its structure in a radical way. The latter takes place, in particular, in the theory of Euclidean time developed by Pestov [6] (in this connection, see also [7]), in the concept of Clifford space–time of Hestenes–Pavsic (see, e.g., [8, 9]), in the framework of 6D geometry proposed by Urusovskii [10], etc.

At a still more fundamental level of consideration, one assumes to derive the geometry of physical space–time from some primordial principle encoding it (perhaps, together with physical dynamics). One can try to relate such an elementary *Code of Nature* with some exceptional symmetry (theory of physical structures of Kulakov [11] and binary geometrophysics of Vladimirov [12]), group or algebra (quaternionic theory of relativity of Yefremov [13] and algebrodynamics of Kassandrov [14, 15]), with algebraically distinguished geometry (Finslerian anisotropic geometry of Bogoslovsky [16] and geometry of polynumbers of Pavlov [17]) as well as with some special "World function" (metrical geometry of Rylov [18]).

Generally, all the above-mentioned and similar approaches affecting the very foundations of physics differ essentially one from another in the character of the first principle (being either purely physical or abstract in nature), in the degree of confidence of their authors to recently predominant paradigms (Lorentz invariance, Standard model, etc.) and in their attitude towards the necessity to reproduce, in the framework of the original approach, the principal notions and mathematical insrumentation of canonical schemes (of Lagrangian formalism, quantization procedure, Minkowski space itself, etc.). In this respect the *neo-Pythagorean* philosophical paradigm professing by the author [19–21] seems most consistent and promising, though difficult in realization.

Accordingly, under construction of an algebraic (logical, numerical) "Theory of Everything" one should *forget all of the known physical theories and even experimental facts* and to unprejudicely *read out* the laws of physical World in the internal properties of some exceptional abstract primordial structure, adding and changing nothing in the course of this for "better correspondence with experiment". In this connection, one should be ready that physical picture of the World arising at the output could have little in common with recently accepted one and that *the real language of Nature* might be quite different from that we have thought out for better description of observable phenomena. In this situation none *principle of correspondence* with former theories could be applied.

We have no opportunity to go into details of the neo-Pythagorean philosophy, quite novel and radical for modern science, sending the reader to [19–21]. Instead, in Section 2 we briefly present its realization in the framework of the "old" version of algebrodynamics developed during the period 1980–2005 [14, 15]. Therein an attempt has been undertaken to obtain the principal equations of physical fields and the properties of particle-like formations as the only consequence of the properties of exceptional quaternion-like algebras, exactly, of the algebra of *biquaternions* $\mathbb{B}$.

We have forcible arguments to regard this attempt successful. From the sole conditions of $\mathbb{B}$ analitivity (generalization of the Cauchy–Riemann equations, see Section 2) we were able to obtain an unexpectedly rich and rather realistic field theory. In particular, as a principal element of the arising picture of the World there turned to be a *flow of light-like rays* densely filling the space and giving rise to a sort of particle-like formations at *caustics* and *focal points*. Such a primordial, matter generating structure has been called the *Flow of Prelight*. From mathematical point of view, this flow is defined by the *twistor structure* of the first equations for biquaternionic field, whereas geometrically it represents itself a *congruence of null rays* of a special type (namely, *shear-free*), below—the *generating congruence*.

Meanwhile, the "geometrical scene" on which the algebraic dynamics displays itself has been, in fact "by hands," restricted to a subspace with canonical Minkowski metric, to ensure the Lorentz invariance of the scheme. Such a procedure was in evident contradiction with the whole philosophy of algebrodynamics, since corresponding subspace does not even form a subalgebra of $\mathbb{B}$ and is thus in no way distinguished in the structure of $\mathbb{B}$ algebra. From a more general viewpoint, neither in our old works nor in those of other authors there has been found any *space–time algebra*, that is, ascertained an algebraic ("numerical") structure which could naturally induce the Minkowski geometry (or include the latter as its part).[1)]

However, in 2005 in [22] we have demonstrated that, under a thorough consideration, the primordial *complex* geometry of $\mathbb{B}$ algebra unavoidably induces a *real* geometry just of the Minkowski type. In this scheme, the additional coordinates of (8D in reals) vector space of $\mathbb{B}$ are naturally compactified and behave like a *geometrical phase* suggesting thus a geometrical explanation of the *wave properties of matter* in general. In the following, this geometry has been called the *phase extension of the Minkowski space*. Its derivation and simplest properties are presented in Section 3.

Discovery of the novel geometry induced by the primordial algebraic struture of complex quaternions[2)], opened wide perspective for construction of a completed version of algebrodynamics [23, 24]. In particular, it turns out that just (and only!) in the primordial complex space there may be realized one of the most interesting and original ideas of Wheeler–Feynman about "all identical electrons as one and the same electron", in its distinct positions on a unique world line. In [23] the set of copies of the sole "generating charge" correlated in dynamics has been called *the ensemble of "duplicons."* We consider geometrodynamical properties of duplicons and related particle-like formations in Section 4.

In Section 5 a naturally arising concept of *complex time* is presented. Indeed, already in the previous version of algebrodynamics (on the real Minkowski background) the temporal coordinate is distinguished in a natural way as an *evolution parameter* of the primordial biquaternionic (and associated twistor) field: the generating Prelight Flow is identified with the flow of time [19, 21]. Now, in the complex pre-space such a parameter unavoidably turns to be two-dimensional, and the related order of sequence of events—indefinite. Thus, in the framework of initially deterministic "classical" theory there arises inevitable *uncertainty of evolution of states* related to effectively stochastic alteration of the evolution parameter itself on the complex plane; we are led, therefore, to accept the concept of complex *random time*. On the other hand, existence of geometrical phase makes it possible to suggest a novel treatment of the phenomena of *quantum interference*, alternative to generally accepted concept of *wave–particle dualism*. In particular, such a treatment relates the notion of the phase of wave function to the classical action of a particle quite in the spirit of Feynman's version of quantum theory. Consideratons of these issues conclude the paper.

2. ALGEBRODYNAMICS OVER MINKOWSKI SPACE

Biquaternionic ($\mathbb{B}$) algebrodynamics is completely based on the (proposed by the author in 1980) version of noncommutative, including biquaternionic, analysis, that is, on generalization of the theory of functions of complex variable to the case of noncommutative algebras of quaternionic type. This version is exposed in detail in the monograph [14] (where one can find references to preceding works) and in the recent review [15].

Essentially, the whole structure of the theory of functions of $\mathbb{B}$ variable $Z \in \mathbb{B}$ follows from invariant definition of a *differential* dF of such, *differentiable in* $\mathbb{B}$, function $F : Z \mapsto Z$ (a direct analog of an analytical function in complex analysis). Specifically, in

[1)] Hestenes was one of the first to consider the concept of space–time algebra [8]. We think, however, that his favourite 16D Dirac algebra cannot in fact be considered in this role since the additional dimensions have no natural physical interpretation.

[2)] The algebra $\mathbb{B}$ is distinguished as a unification of two exceptional (associative with norm and division) algebras, namely of complex numbers and of Hamilton's quaternioins.

account of associativity yet non-commutativity of the algebra one has

$$dF = \Phi * dZ * \Psi, \tag{1}$$

where $\Phi(Z)$, $\Psi(Z)$ are some two auxiliary functions formerly called (left and right) *semi-derivatives* of $F(Z)$.

Relation (1) explicitly generalizes the well-known *Cauchy–Riemann conditions*. Indeed, in the case of commutative algebra of complex numbers it acquires a familiar form

$$dF = F' * dZ, \tag{2}$$

with $F' := \Phi * \Psi$ being an ordinary derivative of an analytical function $F(Z)$ of complex variable Z. Writing (2) down in components one comes to the standard Cauchy–Riemann system of equations. Thus, requirement of invariance of the differential (2) represents itself one of a number of equivalent versions of complex analysis suitable, moreover, for its generalization to a noncommutative case in the form (1). Note that such version was, perhaps, first proposed by Sheffers [25] for construction of the analysis over an arbitrary commutative–associative algebra and nearily after a century used by Vladimirov and Volovich [26] for generalization to superalgebras.

Remarkably, in the case of real Hamilton quaternions $\mathbb{Q}$ the proposed conditions (1) reproduce another exceptional property of complex analysis, namely, the *conformity* of correspondent mapping implemented by any analytical function [27, 28]. However, since the conformal group of Euclidean space $\mathbf{E}^k$ under $k \geq 3$ is finite (exactly, 15-parametrical for $k = 4$), quaternionic analysis built on the base of relation (1), turns to be unattractive in respect of physical applications.

When, however, one passes to the case of $\mathbb{B}$ algebra (i.e., under complexification of $\mathbb{Q}$) *the class of differentiable (in the sense of (1)) functions essentially expands* due to special elements of $\mathbb{B}$—*null divizors* (see details in [14, 15]; corresponding mappings have been called *degenerate conformal*). In this way we naturally come to formulate the first "interpretational" principle:

In the paradigm of $\mathbb{B}$ algebrodynamics there exists a unique fundamental physical field. This is a (essentially complicated and even multivalued) function of $\mathbb{B}$ variable obeying the conditions of $\mathbb{B}$ differentiability (1)—the only primordial "field equations". All of the other "fields" arising in the scheme are secondary and can be defined through (semi)derivatives, contractions, etc. of the input $\mathbb{B}$ field. Their equations also follow from the "master equations" (1).

Realization of this program requires, meanwhile, to resolve the problem of relationship of the 4D complex coordinate space Z, vector space of $\mathbb{B}$ algebra, to the Minkowski physical space–time. As it was already mentioned, correct correspondence between these spaces has been ascertained not long ago in our works and leads to a principally novel view on the geometry of space–time (see below). As to this section, we shall expose only the former version of algebodynamics in which the coordinate space Z is forcibly restricted onto the subspace with Minkowski metric, in order to guarantee the Lorentz invariance of the scheme and to avoid the problem to prescribe a particular meaning to additional "imaginary" coordinates.

Specifically, it is well known that the biquaternion algebra $\mathbb{B}$ is isomorphic to the full 2×2 matrix algebra over $\mathbb{C}$. Further on we shall use the following two equivalent matrix representations of an element $Z \in \mathbb{B}$:

$$Z = \begin{pmatrix} u & w \\ p & v \end{pmatrix} = \begin{pmatrix} z_0 + z_3 & z_1 - iz_2 \\ z_1 + iz_2 & z_0 - z_3 \end{pmatrix} \tag{3}$$

through four complex "null" $\{u, w, p, v\}$ or "Cartesian" $\{z_\mu\}$, $\mu = 0, 1, 2, 3$, coordinates of a biquaternion Z, respectively. Therefore, the 4D complex vector space of $\mathbb{B}$ possesses a natural complex (quasi)metrical form correspondent to the determinant of representative matrix (3),

$$D = z_0^2 - z_1^2 - z_2^2 - z_3^2. \tag{4}$$

This form turns into the Minkowski pseudo-Euclidean metric if only one considers the coordinates $z_\mu \mapsto x_\mu \in \mathbb{R}$ as reals. This corresponds to restriction of a generic matrix (3) to a *Hermitean* one $Z \mapsto X = X^+$. However, we do not intend to restrict, in a similar way, the "field" matrices associated with *functions $F(X)$* of the space–time coordinates $X = \{x_\mu\}$, since the principal physical fields (especially in quantum theory) are considered as complex-valued.

Thus, we come to the second interpretational principle of the considered version of algebrodynamics:

In $\mathbb{B}$ algebrodynamics the coordinate physical space–time is represented by a subspace of the $4\mathbb{C}$ vector space of $\mathbb{B}$ correspondent to Hermitean 2×2 matrices $X = X^+$ with determinant being just the Minkowski metric. After such a restriction the whole algebrodynamical scheme becomes Lorentz invariant by itself.

On a coordinate "cut" correspondent to the Minkowski space first conditions of $\mathbb{B}$ differentiability take the form

$$dF = \Phi * dX * \Psi \tag{5}$$

and represent themselves a sort of "master equations" for some *algebraical field theory* which uniquely determines all its derivable properties. It is also noteworthy that none Lagrangians, commutation relations or other additional structures are used in the theory under consideration. Moreover: system of equations (5) turns to be *overdetermined* and does not allow for any generating Lagrangian structure.

Consequently, overdetermined character of the primordial algebrodynamical relations (5), together with *nonlinearity* of the arising field equations (see below), makes it possible to consider $\mathbb{B}$ algebrodynamics as a *theory of interacting fields* (and "particles" with rigidly fixed, "self-quantized" characteristics, see below).

As for particles, in the classical theory (like algebrodynamics in its original form) in the capacity of those one can obviously take either regular (soliton-like) or singular field formations localized in 3-space. Now we are ready to formulate the last (third) interpretational principle completing the set of first statements of algebrodynamical scheme:

In the framework of $\mathbb{B}$ algebrodynamics, "particles" (particle-like formations) correspond to (point or extended but bounded in 3-space) singularities of the biquaternionic field or its derivatives. The latters may be put into correspondence with singularities of secondary (Maxwell, Yang–Mills, and other) fields associated with any distribution of the primary $\mathbb{B}$ field. Shape, spatial arrangement, characteristics, and temporal dynamics of these particle-like formations are again completely determined by the properties of the master algebrodynamical system of equations for $\mathbb{B}$ field (5).

It should be noted that symmetries (relativistic and conformal among them) of system (5) are considered in detail in the review [15]. Gauge and twistor structures specific for system (5) are also described therein (below we shall return to some of them). Let us now briefly review principal properties and consequences of the $\mathbb{B}$ algebrodynamics (based solely on the conditions of $\mathbb{B}$ differentiability (5)).

1. Each *matrix* component $S(x, y, z, t)$ of a $\mathbb{B}$-differentiable function $F(X)$ satisfies the *complex eikonal equation*

$$\left(\frac{\partial S}{c\partial t}\right)^2 - \left(\frac{\partial S}{\partial x}\right)^2 - \left(\frac{\partial S}{\partial y}\right)^2 - \left(\frac{\partial S}{\partial z}\right)^2 = 0. \quad (6)$$

This nonlinear, Lorentz and conformal invariant equation substitutes the Laplace equation in complex analysis and form the basis of the algebraic field theory.

2. Primary conditions (5) can be reduced to a simpler system of equations of the form

$$d\xi = \Phi dX \xi \quad (7)$$

for effectively "interacting" 2-spinor field $\xi(X) = \{\xi_A\}$, $A = 1, 2$, and potentials $\Phi(X) = \{\Phi_{AA'}\}$ of a complex gauge-like field (see for details [15].

3. Integrability conditions of reduced overdetermined system (7) are just the *self-duality conditions*

$$F = iF^* \quad (8)$$

for the field strengths of gauge potentials $\Phi_{AA'}$. Consequently, complexified Maxwell and $SL(2,\mathbb{C})$ Yang–Mills free equations are both satisfied on the solutions of master system (7).

4. A field of a *null 4-vector* k_μ: $k^\mu k_\mu = 0$ can be constructed from fundamental 2-spinor $\xi(X)$ as follows:

$$k_\mu = \xi^+ \sigma_\mu \xi, \quad (9)$$

where $\sigma_\mu = \{I, \sigma_a\}$, $a = 1, 2, 3$, is the canonical basis of 2×2 matrices. As a consequence of master system (7), the null *congruence* of rays tangent to k_μ is rectilinear (geodetic) and *shear-free*. This congruence of rays plays an extremely important role in algebrodynamics; below we shall call it *generating congruence*[3)]. In the context of algebrodynamics it is important that an effective Riemannian metric $g_{\mu\nu}$ of a special form

$$g_{\mu\nu} = \eta_{\mu\nu} + h(X) k_\mu k_\nu \quad (10)$$

(the so-called Kerr–Schild metric [32]) may be put in correspondence with any $\mathbb{B}$ field or associated generating congruence. This is a deformation of the flat Minkowski metric $\eta_{\mu\nu}$ preserving all the defining properties of generating congruence. Note that a self-consistent algebrodynamical scheme over a *curved* space–time background has been developed in [33].

5. In contrast to ordinary nonlinear field models, in the algebrodynamics it turns to be possible to obtain *general solution* of the master system of equations (7) or (5) in an implicit *algebraic* form. The procedure is based upon the (well-known in GTR) *Kerr theorem* [32, 34] that gives full discription of null shear-free congruences on the Minkowski or Kerr–Schild background, and makes also use of a natural generalization of this theorem [35, 36], namely, of general solution of the complex eikonal equation obtained therein. Briefly, the procedure of searching

[3)]Congruences like these naturally arise in the framework of GTR and were widely studied, in particular, by Newman [29], Kerr [30], and Burinskii [31].

the solutions of eikonal equation and associated congruence can be desribed as follows (for details see [15, 36]).

Using gauge (projective) symmetry, one reduces fundamental spinor $\xi(X)$ to the ratio of its two components choosing, say,

$$\xi^T = (1,\ g(X)); \tag{11}$$

then *any solution of the algebrodynamical field theory is defined via the only complex function* $g(x,y,z,t)$—component of the *projective* 2-spinor ξ.

In turn, any solution for $g(X)$ is obtained in the following way. Consider an arbitrary (almost everywhere smooth) surface in the *3D complex projective space* $\mathbb{C}P^3$; it may be set by an algebraic constraint of the form

$$\Pi(g, \tau^1, \tau^2) = 0, \tag{12}$$

where $\Pi(\dots)$ is an arbitrary (holomorphic) function of three complex arguments. Let now these latters be linearily linked with the points of Minkowski space through the so-called *incidence relation* [34]

$$\tau = X\xi \tag{13}$$

or, in components,

$$\tau^1 = wg + u,\ \ \tau^2 = vg + p, \tag{14}$$

where in the considered case of real Minkowski space the coordinates (3) $u, v = ct \pm z$ are real and $p, w = x \pm iy$ complex conjugated. It is known that two spinors ξ, τ related with points X via the incidence relation (13) or (14) form the so-called *projective twistor* of the Minkowski space [34].

After substitution of (14) into equation of generating surface (12) the latter acquires the form of an algebraic equation

$$\Pi(g, wg + u, vg + p) = 0 \tag{15}$$

with respect to the only unknown g, whereas the coordinates $\{u, v, p, w\}$ play the role of parameters. Resolving the equation above at each point of the Minkowski space X, one obtains some (generally multivalued) field distribution $g(X)$.

In a rather puzzling way (the proof may be found, say, in [37, 38]), *for any generating function* Π *and any continious branch of the solution under consideration the field* $g(X)$ *identically satisfies both fundamental relativistic equations—linear wave equation* $\Box g = 0$ *and nonlinear equation of complex eikonal (6)*. Correspondent spinor ξ (in the gauge (11)) satisfies meanwhile the equations of shear-free null congruences and, according to the above-mentioned Kerr theorem, all such congruences can be obtained with the help of the exposed algebraic procedure.

6. It has been demonstrated in [39, 40] that the complexified *electromagnetic field* associated with fundamental spinor ξ (it identically satisfied self-duality conditions (8) and, thus, the homogeneous Maxwell equations) can be directly expressed through the function g (obtained as a solution of algebraic constraint (15)) and its derivatives $\{\Pi_A, \Pi_{AB}\}$ with respect to the twistor arguments $\{\tau^A\},\ A = 1, 2$. Specfically, for *spintensor* of electromagnetic field strength φ_{AB} one gets

$$\varphi_{AB} = \frac{1}{P}\left\{\Pi_{AB} - \frac{d}{dg}\left(\frac{\Pi_A \Pi_B}{P}\right)\right\}, \tag{16}$$

with $P := d\Pi/dg$. Strengths of the associated Yang–Mills field can also be represented algebraically via (16) and the spinor g itself.

7. It can be seen from representation (16) that the electromagnetic field strength turns to infinity at the points defined by the condition

$$P = \frac{d\Pi}{dg} \equiv \frac{\partial \Pi}{\partial g} + w\Pi_1 + v\Pi_2 = 0. \tag{17}$$

Similar situation takes place for singularities of associated Yang–Mills field and the *curvature field* of effective Kerr–Schild metric (10) (see [31, 32, 37]). Therefore, in the context of $\mathbb{B}$ algebrodynamics one is brought to identify *particles with locus of common singularities of the curvature and gauge fields*. It is also reasonable to assume under this identification that, instantaneously, particle-like singularities are bounded in 3-space.[4)]

8. With respect to the primary $\mathbb{C}$ field $g(X)$ obtained from constraint (15), condition (17) defines its *branching points*. Geometrically, this corresponds to *caustics* of the light-like rays of generating congruence. Generally speaking, instead of the primary $\mathbb{B}$ field and correspondent multivalued field $g(X)$ one can equivalently consider the fundamental congruence consisting, generically, of a (great) number of individual branches ("subcongruences" [36]) and forming caustics-particles at the points of merging of rays from some two of them, i.e., at the *envelope*. This all-matter-generating primordial structure in [19, 21, 36] has been called the *prelight flow*, or the "Prelight."

9. At the same time, existence of the Prelight flow immediately distinguishes the temporal structure[5)]. Indeed, incidence relation (13) preserves its form under a one-parametrical coordinate transformations of the form

$$x_a \mapsto x_a + n_a s, \quad t \mapsto t + s, \tag{18}$$

[4)]For string-like singularities expanding to infinity and found, e.g., in [37] another interpretation is needed (cosmic strings, etc.).

[5)]Actually, this is true for any twistor structure in general.

$$n_a n_a = 1, \quad s \in \mathbb{R},$$

corresponding to a translation in 3-space along each of rectilinear rays of the congruence, i.e., along spatial directions specified by the unit vector $\mathbf{n}$ defined as

$$\mathbf{n} = \frac{\xi^+ \sigma \xi}{\xi^+ \xi} \tag{19}$$
$$= \frac{1}{1+gg^*}\{g+g^*, i(g-g^*), 1-gg^*\}.$$

Under such transformations physically correspondent to the process of *propagation* of the principal field with universal velocity $V = c = 1$, all of the three components of the projective twistor are preserved in value, as well as the direction vector $\mathbf{n}$ itself. Then, in accord with representation (18), one can regard these transformation as a prototype of the *course of time* and the Prelight Flow itself as the *Time Flow*. In more details these issues were considered in [21, 36], and in Section 5 we shall see in what an interesting way they are refracted under introduction of complex prespace.

10. Particles identified with common singularities of gauge and curvature fields exhibit a number of remarkable properties specific for real matter constituents. The most interesting is, perhaps, that of *self-quantization* of electric charge. This property follows from over-determinance of master system (7) and self-duality of associated field strength (8) and is, partially, of topological origin. According to the quantization theorem proved in [41]), for any isolated and bounded (i.e., particle-like) singularity of electromagnetic field (16) electric charge is either null or necessarily integer multiple to some minimal, *elementary* value, namely, to the charge of fundamental static solution to $\mathbb{B}$ equations (7). The latter is a direct analog of the well-known Kerr–Newman solution in GTR. It follows from twistor constraint (12) with generating function Π of the form

$$\Pi = g\tau_1 - \tau_2 + 2iag = wg^2 \tag{20}$$
$$+ 2(z+ia)g - p = 0, \quad z := (u-v)/2,$$

resolving which one obtains the two-valued solution

$$g = \frac{p}{z \pm r} \equiv \frac{x+iy}{z+ia \pm \sqrt{x^2+y^2+(z+ia)^2}}, \tag{21}$$
$$a \in \mathbb{R}.$$

With the above solution one can associate the famous *Kerr congruence* with caustic of the form of a singular ring of radius a correspondent to the locus of branching points of function (21). Particularly, in degenerate case $a = 0$ of a point-like singularity the associated via (16) electric field is the Coulomb one but electric charge q of singularity is strictly fixed in absolute value (in the accepted normalization $q = \pm 1/4$) [14, 28]. Correspondent effective metric (10) is just the Reissner–Nördstrem solution of the Einstein–Maxwell equations.

In a general case $a \neq 0$ solution (21) leads to the field and metric exactly correspondent (under additional requirement on electric charge to be unit!) to the above-mentioned Kerr–Newman solution (in the regime of a naked singularity free of horizon). Carter [42] was the first to notice that correspondent gyromagnetic ratio for this field distribution is exactly equal to its anomalous value for Dirac fermion. This stimulated subsequent studies (of Lopez, Israel, Burinskii, Newman, *et al.*) in which the Kerr singular ring, with associated set of fields, has been regarded as a *model of electron*. Note that in the algebrodynamical scheme this consideration is still more justified since the electric charge therein is necessarily fixed in modulus and may be identified as the elementary one. Thus,

In the framework of $\mathbb{B}$ algebrodynamics over Minkowski space the electron can be represented by the Kerr singular ring (of Compton size) related to a unique static axisymmetrical solution (21) of equations (7), or of the constraint (15).

11. A number of other exact solutions of the initial algebrodynamical equations and of related eikonal, Maxwell, and Yang–Mills equations have been obtained in [36, 41], among them a *bisingular* solution and its toroidal modification [38]. They correspond to generating function Π in (15) quadratic in g. More complicated solutions demand the computer assistance for solving the algebraic relation (15). However, the (most interesting) structure of singular loci of these distributions can be determined through elimination of the unknown g from the set of two algebraic equations (15) and (17)[6]. The complex equation arising under the procedure

$$\Pi(x,y,z,t) = 0 \tag{22}$$

represents itself the *equation of motion* of particles-singularities and, moreover, at a fixed instant fixes their spatial distribution and shape. In this way, we have examined the structure of singularities for complicated solutions of master equations and associated biquaternionic and electromagnetic fields. As for the latter, there has been obtained a peculiar solution to free Maxwell equations (!) describing the *process of annihilation* of two unlike (and necessarily unit) charges, with accompaning radiation of a singular

[6] In the case of polinomial form of generating function Π the procedure reduces to determination of the *resultant* of two polinoms and can be easily algorithmized.

wave front [36], a class of the wave-like singular solutions [40], etc.

12. If one restricts itself by *generic* solutions to master equations (7) or to associated Maxwell equations[7)], then their singular locus will (instantaneously) represent itself as *a number of one-dimensional curves—"strings."* Generally, these strings (though neutral or carrying unit charges) are *unstable* in shape and size with respect to a small variation of parameters of the generating function Π. As an example, consider a special *deformation of the Kerr solution and congruence* [3] defined by the following modification of the Kerr generating function (20):

$$\Pi = g\tau^1(1-ih) - \tau^2(1+ih) + 2iag, \qquad (23)$$

in which the parameter $h \in \mathbb{R}$ enters in addition to the standard Kerr parameter $a \in \mathbb{R}$. As a result, from the constraint $\Pi = 0$ one obtains a novel solution for function g that defines still axisymmetrical but now time-dependent generating congruence of rays. Its caustic defined by the branching points of g is represented by a uniformly collapsing into a point and, afterwards, expanding to infinity *singular ring*:

$$z = 0, \quad \rho := \sqrt{x^2+y^2} = v(t-t_0), \qquad (24)$$

where $t_0 = a/\sqrt{1+h^2}$ and velocity of collapse/expansion $v = h/\sqrt{1+h^2}$ is always less than the light one $c = 1$. Thus,

The Kerr congruence is unstable with respect to a small perturbation of controlling parameters of the generating function. This lets one expect also the instability of the Kerr (Kerr–Newman) solution of (electro) vacuum Einstein equations, since the latter is defined, to a considerable degree, by the structure of null congruence of the above-presented type.

Note in addition that the deformed ring still carries a fixed elementary charge but cannot escape, nonetheless, being radiated to infinity.

At this point we complete our brief review of the "old" algebrodynamics on the Minkowski background by the following remarks. In fact, from a single initial condition of $\mathbb{B}$ differentiability we were able to develop a self-consistent theory of fields and particle-like formations possessing a whole set of unique and physically realistic properties. The only *ad hoc* assumption made during the construction of algebrodynamical theory, in order to ensure its Lorentz invariance, was a rather artificial restriction of the coordinate 4ℂ vector space of $\mathbb{B}$ algebra onto the subspace with Minkowski metrical form. On the other hand, the structure of *string-like* singularities-particles arising on $\mathbf{M}$ under this procedure turns to be unstable and, perhaps, diffuses with time. Together, these considerations suggest the necessity of a more successive analysis of the geometry "hidden" in the algebraic structure of biquaternion algebra, and of probable links of its 4ℂ vector space with the true physical geometry. On this way we immediately discover a completely novel geometry of (extended) space–time presented in the next section.

[7)]That is, by solutions free of any symmetry, in particular, nonstatic and nonaxisymmetric.

3. BIQUATERNION GEOMETRY AND PHASE EXTENSION OF THE MINKOWSKI SPACE

Let us return to matrix representation (3) of the elements $Z \in \mathbb{B}$ of biquaternion algebra. Restriction to *unitary* matrices $Z \mapsto U : U^+ = U^{-1} * \det U$ reduces the algebra $\mathbb{B}$ to that of real Hamilton quaternions $\mathbb{Q}$. Remind that $\mathbb{Q}$ is one of the two *exceptional associative division algebras*, together with complex algebra. Transformations preserving, together with unitarity, the structure of multiplication in $\mathbb{Q}$ (inner automorphisms) are of the form

$$U \mapsto S * U * S^{-1}, \quad S^{-1} = S^+, \qquad (25)$$
$$S \in SU(2).$$

Under these, the diagonal (real) component of a matrix U is invariant whereas the other three $\{x_1, x_2, x_3\}$ behave as components of a rotating 3-vector (note that both $\pm S$ correspond to the same rotation: spinor structure). So the automorphism group of quaternion algebra $Aut(\mathbb{Q}) = SU(2) \cong SO(3)$ is $2:1$ isomorphic to the group of 3D rotations, with the main invariant

$$l = x_1^2 + x_2^2 + x_3^2, \qquad (26)$$

defining *Euclidean* structure of geometry induced by the algebra $\mathbb{Q}$. In this sense, from the times of Hamilton, *exceptional algebra of real quaternions is considered as the algebra of physical background space* and, in the algebrodynamical paradigm, *predetermines its dimensionality and observable Euclidean structure*.

We can then apply the same "Hamilton logic" to the algebra of biquaternions $\mathbb{B}$. Now the elements $Z \in \mathbb{B}$ are represented by complex matrices of *generic* type (3), and multiplication in $\mathbb{B}$ is preserved under transformations

$$Z \mapsto M * Z * M^{-1}, \quad \det M = 1, \qquad (27)$$
$$M \in SL(2,\mathbb{C}).$$

In full analogy with the real case, the diagonal component z_0 in (3) remains invariant, and the three others $\mathbf{z} = \{z_1, z_2, z_3\}$ manifest themselves as a 3D *complex* vector under *complex* rotations. Thus, one has: $Aut(\mathbb{B}) = SL(2,\mathbb{C}) \cong SO(3,\mathbb{C})$.

Some explanations must be presented at this point. It is well known that the 6D (in reals) group $SL(2,\mathbb{C})$ is a covering of the Lorentz group realizing its spinorial representation; the same is true for the $2:1$ isomorphic group of 3D complex rotations $SO(3,\mathbb{C})$. Specifically, Lorentz transformations can be represented in the form analogous to (27),

$$X \mapsto M * X * M^+, \tag{28}$$

but act on the subspace $Z \mapsto X$ of *Hermitean* matrices $X = X^+$ with determinant representing the Minkowski metric. It is just this restriction that we considered in the previous section. Now, however, we are interested in natural geometry induced by the *full* structure of 8D (in reals) vector space Z of $\mathbb{B}$ algebra, in its hypothetical relations to the Minkowski space and in physical meaning of four additional coordinates. It should be noted that, surprisingly, this geometry has not been discovered until now. As we shall see, corresponding construction is rather transparent and successive.

We have seen that the structure of $\mathbb{B}$ multiplication is preserved under 3D complex rotations forming the $SO(3,\mathbb{C})$ group. The main *complex* invariant of these transformations, the analog of Euclidean invariant (26) of the real algebra $\mathbb{Q}$, is represented by a *holomorphic* (quasi)metrical bilinear form

$$\sigma = z_1^2 + z_2^2 + z_3^2 \equiv |\mathbf{z}|^2, \tag{29}$$

the (squared) "complex length" of a vector $\mathbf{z}$. It should be emphasized that all other metrical forms, the Hermitean metric among them, that could be canonically defined on the vector space $\mathbb{C}^4$ itself (or on its subspace $\mathbb{C}^3$) are, in fact, meaningless in the framework of the algebrodynamics since *they do not preserve their structure under* $\mathbb{B}$ *automorphisms*.

On the other hand, from complex invariant (29) one can naturally extract a positive definite (exactly, non-negative) *Finslerian* metrical form of the fourth degree taking the square of complex modulus of the considered invariant

$$S^2 := \sigma\sigma^* = |\mathbf{z}|^2|\mathbf{z}^*|^2. \tag{30}$$

As the next step, one can make use of the following remarkable identity (see, e.g., [43]):

$$|\mathbf{z}|^2|\mathbf{z}^*|^2 \equiv (\mathbf{z}\cdot\mathbf{z}^*)^2 - |i\mathbf{z}\times\mathbf{z}^*|^2 \tag{31}$$

that can be explicitly verified. Taking it into account, one can represent the positive-definite invariant (30) in the form of a Minkowski-like interval [22]:

$$S^2 = T^2 - |\mathbf{R}|^2 \geq 0, \tag{32}$$

in which the quantities T and $\mathbf{R}$ defined through the scalar $(\cdot)$ and vector $(\times)$ products of complex 3-vectors as

$$T := \mathbf{z}\cdot\mathbf{z}^*, \quad \mathbf{R} := i\,\mathbf{z}\times\mathbf{z}^*, \tag{33}$$

aquire, respectively, the meaning of temporal and spatial coordinates of some effective 4D space with a Minkowski-type metric. Note also that such an identification is quite unformal since under the $\mathbb{B}$ automorphisms acting as 3D complex rotations the quantities $T, \mathbf{R}$ *transform one through the others just as the temporal and spatial coordinates do under Lorentz transformations*.[8)]

Thus, *the main real invariant of biquaternion algebra, being positive definite, induces nonetheless the structure of causal domain of the Minkowski space* correspondent to the interior of the light cone (together with its light-like boundary). In this scheme, the events that are not causally connected as if do not exist at all (just as this should be from a successive viewpoint of STR). We come, therefore, to a paradoxical but much interesting, both from physical and mathematical viewpoints, concept of the *physical space—time with positive definite metric*.

Consider now the *phase* part of complex invariant (29). The latter can be represented in the form

$$\sigma = S\exp^{i\alpha}, \tag{34}$$

with absolute value S correspondent to the Minkowski interval and the *phase* α also invariant under 3D complex rotations (that is, in fact, under Lorentz transformations). In this connection, the *noncompact* (corresponding to modulus) part of the initial invariant is responsible for *macrogeometry* explicitly fixed by an observer: remarkably, it turns to be exactly of a Minkowski type. At the same time, its phase, *compact* part determines geometry of the "fiber" and, perhaps, reveals itself at a *microlevel* being, in particular, related to universal *wave* properties of matter (see Section 5). In the other respect, invariant α has the meaning of the *phase of proper complex time* as this can be seen from (34) and will be discussed below.

Thus, we accept a novel concept of the background space—time geometry as of *the phase extension of (a causal part) of the Minkowski space* predetemined by the initial complex-quaternionic structure, with coordinates *bilinear* in those of the primordial and "actually existing" $\mathbb{C}^3$ space.

Note that in literature dealing with different versions of complex extensions of space—time (see, e.g., [44, 45]) one usually encounters the procedure of separation of complex coordinates into real "physical" and imaginary "unphysical" parts (alternative to their separation into "modulus" and "phase" parts in our approach). This procedure is, actually, inconsistent,

8) In fact, these transformations have some peculiarities in comparison with canonical Lorentz transformations, see [22] for details.

since both parts are completely equivalent in internal properties and should equally contribute to the induced real geometry one constructs.

Nonetheless, the above-mentioned *linear* separation of complex coordinates is rather demonstrative. Specifically, consider *a couple of 3D real vectors* $\{\mathbf{p}, \mathbf{q}\}$ associated with a complex vector $\mathbf{z}$:

$$\mathbf{z} = \mathbf{p} + i\mathbf{q}. \quad (35)$$

In this representation the principal invariant (29) takes the form

$$\sigma = (|\mathbf{p}|^2 - |\mathbf{q}|^2) + i(2\mathbf{p} \cdot \mathbf{q}), \quad (36)$$

and corresponds to a pair of invariants in which one easily recognizes the two well-known *invariants of electromagnetic field* (with vectors $\mathbf{p}$, $\mathbf{q}$ identified as the field strengths of electric and magnetic field, respectively). The noticed analogy of complex coordinates and electromagnetic field seems much suggesting and requires thorough analysis.

Express now through vectors $\mathbf{p}$, $\mathbf{q}$ the effective temporal and spatial coordinates (33):

$$T = |\mathbf{p}|^2 + |\mathbf{q}|^2, \quad \mathbf{R} = 2\mathbf{p} \times \mathbf{q} \quad (37)$$

and note that the temporal coordinate is positive definite (in Section 5 we shall relate this property with *time irreversibility*). As to three spatial coordinates, they form an *axial* vector so that the choice of sign corresponds to reference frame of definite chirality.

Finally, let us write down a remarkable relation [22] that links the module V of the velocity $\mathbf{V} = \delta\mathbf{R}/\delta T$ of motion of a material point in the induced Minkowski space with characteristics of initial complex space $\mathbb{C}^3$, namely, with invariant phase α and the *angle* θ between vectors $\mathbf{p}$ and $\mathbf{q}$:

$$\cos^2\theta = \frac{1 - V^2}{1 + V^2 \coth^2\alpha}. \quad (38)$$

In a limited case of motion with fundamental velocity $V = c = 1$ one gets $\theta = \pi/2$, so that vectors $\mathbf{p}$, $\mathbf{q}$ are orthogonal to each other and to the direction of motion $\mathbf{V}$ (in analogy with electromagnetic wave). From (37) one obtains also that in this "light-like" case the two vectors are equal in modulus, invariant σ turns to zero, and the phase α becomes indefinite.

In the opposite case of a "particle" at rest $V = 0$ one gets $\theta = 0, \pi$, so that two different ("para" and "ortho") relative orientations of vectors $\mathbf{p}$, $\mathbf{q}$ are possible. This remarkable property might be related to two admissible projections of the *spin vector* onto an abitrary direction in 3-space.

4. COMPLEX ALGEBRODYNAMICS AND THE ENSEMBLE OF "DUPLICONS"

According to the first principles of algebrodynamical approach, the true dynamics takes place just in the biquaternionic "pre-space" $\mathbb{C}^4$. In fact, we are able to explicitly observe only a "shadow" of this primordial dynamics on the induced (via mapping (33)) real Minkowski-like space with additional phase and causal structures.

As another ground under construction of *complex algebrodynamics* there serves a distinguished role of the "complex null cone", a direct analog of the real Minkowski light cone. Specifically, consider two points $P, P^{(0)} \in \mathbb{C}^4$ with coordinates Z connected through algebraic relation of the form

$$[z_1 - z_1^{(0)}]^2 + [z_2 - z_2^{(0)}]^2 + [z_3 - z_3^{(0)}]^2 = [z_0 - z_0^{(0)}]^2. \quad (39)$$

Then it is easy to demonstrate [23] using the incidence relation $\tau = Z\xi$ (comp. with real case (13)) that the twistor field (as well as the principal spinor field g and the initial biquaternionic field) *takes equal values in all the points of the complex null line connecting these points*, that is, along an *element* of the null cone. In this respect the position and displacement of such points are correlated. It is noteworthy that in both sides of *null cone equation* (39) there stands one of the two fundamental invariants of $\mathbb{B}$ algebra so that *the primordial complex geometry dynamically reduces to the geometry of smaller space* $\mathbb{C}^3$ *with holomorphic (quasi)metrical form (29)*. As we are already aware of, this gives rise to real effective geometry of the Minkowski type. Note also that for a fixed value of (the two equal) invariants equation (39) defines a complex 2-sphere. The latter manifold is $SO(3,\mathbb{C})$-invariant and closely related to unitary representations of the Lorentz group [46].

Finally, in full analogy with the "old" version of algebrodynamics on $\mathbf{M}$, let us *identify particles with singularities of the biquaternionic and associated fields, geometrically—with caustics of generating congruence*. In this connection, recall that generic singularities on $\mathbf{M}$ have the structure of one-dimensional curves—"strings". However, on the complexified $\mathbb{C}^3$ background singularities manifest much richer structure.

Let us also emphasize from the beginning that the dynamical principles of complex algebrodynamics are completely the same as of its old version on $\mathbf{M}$. Specifically, we only make use of biquaternionic fields obeying the $\mathbb{B}$-analyticity conditions or, equivalently, of twistor fields defining a *generating congruence* of complex null rays with zero shear. In particular, all the rules of definition of the set of relativistic fields (Section 2) do not require any modification in the complex case.

Consider now generating congruences (and, correspondingly, biquaternionic, twistor, and associated gauge fields) of a special and physically interesting type. These are congruences with a *focal line*—a world line of some virtual point charge "moving" in complex extension $\mathbb{C}^4$ of the Minkowski space.[9] Structures like this have been first considered in the framework of GTR by Newman [29, 47]; further on, congruences with a focal line will be called *Newman's congruences*.[10]

In this connection, consider a point-like singularity-particle "moving" in complex space $\mathbb{C}^4$ along a "trajectory" $z_\mu = z_\mu(\tau)$, $\tau \in \mathbb{C}$, $\mu = 0, 1, 2, 3$. Points, where the primordial twistor (spinor) field takes the same values as in the vicinity of the "particle," are defined by null cone equation (39). However, let these points belong themselves to the considered world line and represent thus other "particles". Then null cone equation (39) acquires the form

$$L := [z_1(\lambda) - z_1(\tau)]^2 + [z_2(\lambda) - z_2(\tau)]^2 + [z_3(\lambda) - z_3(\tau)]^2 - [z_0(\lambda) - z_0(\tau)]^2 = 0 \qquad (40)$$

and for any τ has, in general, a great (or even infinite) number of roots $\lambda = \lambda^{(n)}(\tau)$ defining an ensemble of correspondent "particles" $z_\mu^{(n)}$. These are arranged in various points of the same complex world line and dynamically correlated ("interact") with the initial (generating) particle.

Such a set of "copies" of a sole point-like particle "observing itself" (both in its past and future, see Section 5) was first considered in our works [23, 24] and was called therein the ensemble of *duplicons*. It is noteworthy that on real Minkowski background equation (40) (which on $\mathbf{M}$ turns to be an ordinary *retardation equation*), in the case when the point of observation belongs itself to the world line of a particle, has a unique solution independently of the form of a trajectory, namely, the trivial solution $\lambda = \tau$. Thus, the concept of duplicons cannot be realized on the background of ordinary Minkowski space $\mathbf{M}$.

In [23, 24] we were guided by the old idea of R. Feynman and J.A. Wheeler[11] and considered each duplicon in the capacity of an *electron model*. Indeed, in full correspondence with Feynman—Wheeler conjecture, in the arising picture "all of the electrons" are, essentially, "one and the same electron" in various locations on a unique world line. In fact, however, the arising structure of singularities suggests a more natural, though exotic, interpretation.

Indeed, let us consider a primordial "generating" duplicon in the capacity of an "elementary observer" **O**.[12] All other duplicons on its null cone (40), though dynamically correlated with **O**, are in fact "invisible" and not perceived by the elementary observer: *any "signal" is absent*! It is thus natural to conjecture that the act of "perception" (actually—of *interaction*) takes place when only a null complex line (an element of the complex null cone) connecting **O** with some duplicon becomes "material", that is,—a *caustic* of the generating congruence.

It is easy to determine the caustic locus of the congruence from null cone equation (39). Similar to the case of general solution (15) of the $\mathbb{B}$-analiticity equations, in the case of a Newman's congruence caustics coincide with the *branching points* of the principal spinor field g or, equivalently, of the field of *local time* of a duplicon $\tau(Z)$.[13] At these points one observes the *amplification* of the principal twistor-biquaternionic field (preserved along the elements of the complex null cone) that can be regarded as the process of propagation of a "signal" to (from) the observer **O**, see below.

In turn, branching points correspond to *multiple roots* of the null cone equation defined by the condition

$$L' := -\frac{1}{2}\frac{dL}{d\lambda} = z'_a(z_a(\lambda) - z_a(\tau)) - z'_0(z_0(\lambda) - z_0(\tau)) = 0 \qquad (41)$$

(summation over $a = 1, 2, 3$ is assumed and prime denotes differentiation by λ). Together with initial defining relation for duplicons (40), the above condition specifies a *disrete set of positions of the observer (via its local times $\tau = \tau^{(k)}$) and of a pair of duplicons joining at a correspondent instant (defined as one of mupliple roots $\lambda = \lambda^{(k)}$)*. Thus, elementary *interaction act* can be regarded as a *fusion of some two duplicons* (a, b) (with $\lambda(a) = \lambda(b)$) considered with respect to the observer **O** at some of its positions (with $\tau = \tau^{(k)}$). At such instants

[9] Exact definition and specification of such congruences is presented, say, in [23].

[10] At present, this approach is intensively developed by Newman himself with collaborators [48] as well as by Burinskii [49].

[11] Their below presented construction, by virtue of the abovementioned reason, cannot be realized on real $\mathbf{M}$.

[12] To model a real *macroscopic* observer, instead of a trajectory of an individual duplicon $z_\mu(\tau)$, one should introduce some *averaged* trajectory simplest of which is represented by a null complex line and, under its mapping into real Minkowski space, corresponds to uniform rectilinear motion of an inertial observer.

[13] This field satisfies the complex eikonal equation [23]. On the real Minkowski background this is analogous to the field of the "retarded" time.

a process of the field amplification occurs along a null complex line—a caustic—connecting the observer and the two coinciding duplicons. As it was already mentioned, under its mapping into $\mathbf{M}$ this line corresponds to some rectilinear path of a field *perturbation* moving in uniform with velocity $V \le c$.

We are now in a position to naturally distinguish particles-singularities in the scheme under consideration as *matter constituents* and *interaction carriers*, in full analogy with generally accepted theoretical classification. First of them form the ensemble of identical duplicons and can move along a very complicated and mutually concordant trajectories, geometrically—along the *focal curve of generating congruence*. As to the second, they always move along rectilinear line elements of the complex null cone connecting a pair of "interacting" duplicons. In this process, the two merging duplicons represent an entire particle (see below) and stand for an *emitter*, whereas the observer—for a *detector* of propagating "signal"; the problem of temporal ordering arising in this connection will be discussed in Section 5.

Thus, we are led to the conclusion that any *elementary object (electron?)* may be fixed by an observer only at some particular instants and *represents itself a pair of pre-elements—duplicons—emmiting a signal towards the observer when and only when their positions coincide in (complex) space*. At all the rest time these pre-elements–duplicons–are separated in space, do not radiate, and, consequently, can be detected by none observer.[14)]

Conjecture about *duplicons as halves of the electron* revealing themselves solely at the instants of pairwise fusion strongly correlates with modern concept and observations of *fractional charge* (see, e.g., the review [50]) and, on the other hand, makes it possible to offer an alternative explanation for the wave properties of microobjects, particularly, for the *quantum interference phenomena*.

Indeed, let a pair of duplicons be identified as an electron at an instant of the first fusion, via the caustic-signal emitted towards an observer. In the following, these "twins" diverge in space and, in particular, can pass through different "slots" in an idealized interference experiment. As a result, they can again reveal themselves only at an instant of the next fusion accompanied by a new act of emission of a signal-caustic in the direction of the observer. Between the two fusions, each of the "twins" acquires a particular *phase lag*, namely, of geometrical phase α of the principal complex invariant (34). Since, however, the complex coordinates of both "twins" at the instant of fusion should be equal, for the acquired phase lag one has

$$\Delta\alpha = 2\pi N, \quad N = 0, \pm 1, \pm 2, \ldots \qquad (42)$$

Thus, *there exists only a certain set of points at which a microobject might be once more observed after some its primary "registration"*. This stronly resembles the well-known procedure of *preparation* of a quantum-mechanical state and of the following *QM measurement*, respectively. However, in the above presented picture we do not encounter any sort of the *wave–particle dualism*, of the de Broglie wave, etc. Each matter pre-element—duplicon—manifests itself as a typical point-like corpuscular, whereas phase relations are of a completely geometrical nature and relate to some internal space of a "fiber" over $\mathbf{M}$[15)]. Below we shall once more return to discuss the interference phenomenon.

5. RANDOM COMPLEX TIME AND QUANTUM UNCERTAINTY

Essentially, it is meaningless to discuss the problem of dynamics in complex space before one specifies the notion of *complex time*. In fact, we have already seen that the *evolution parameter* $\tau \in \mathbb{C}$ of an "elementary observer" $\mathbf{O}$, that is, the parameter of the "world line" $z_\mu(\tau)$ of a generating point singularity is now complex-valued. This means that subsequent position of the observer on its "trajectory" (under $\tau \mapsto \tau + d\tau$) is *indefinite* by virtue of arbitrariness of alteration of the *phase* of parameter τ.

On the one hand, any value of τ one-to-one corresponds to a certain position of the observer $\mathbf{O}$ (and of associated set of duplicons correlated with $\mathbf{O}$ through the null cone constraint) and, therefore, to *a definite "state of the Universe"* with respect to a given observer.

On the other hand, a particular *realization* of those or other continuations of the trajectory is ambiguous being ruled by an unknown law of "walk" of the evolution parameter τ across the complex plane, that is, by the form of a curve $\tau = \tau(t)$ with monotonically increasing real-valued parameter $t \in \mathbb{R}$.[16)] In [23, 24] this was called the *evolution curve*.

[14)] In the complex algebrodynamics there exists also another class of singularities representing themselves as a sort of "three-element" formations. One can speculate about their probable relation to the quark content of matter.

[15)] In our scheme, the fiber itself defines the structure of effective Minkowski base. Such situation is unique and, in paticular, can find application in the theory of Calabi–Yau manifolds with a $3\mathbb{C}$ fiber structure.

[16)] One can evidently represent this parameter by the length of the curve.

Note that only after specification of the form of evolution curve one can ascertain the *order* of succession of events and even distinguish past from future. It is just this form that defines the *time arrow* and predetermines, in particular, which of the (completely identical in dynamics) duplicons is "younger" and which "older" than a certain "elementary observer" **O**.

In the framework of neo-Pythagorean ideology of algebrodynamics, the form of the universal evolution curve should follow from some general mathematical considerations and be exceptional with respect to its internal properties; unfortunately, at present the form is unknown. Up to now it only seems natural to expect that this "Time Curve" is extremely complicated and entangled (being, probably, of a fractal-like nature). Where this is the case, *for us* the character of alteration of the evolution parameter on its complex plane would effectively represent itself a *random walk*. Moreover, one may conjecture that this walk is *discrete* whereas the generating worldline $z_\mu(\tau)$ itself remains complex analytical: these two are completely independent. Then in the scheme there arises the *time quanta*—"chronon". We shall see below that it has to be of order of the Compton size, not of the Plank one. From different viewpoints the latter concept has been advocated in a number of works (see, e.g., [51] and references therein).

It is noteworthy that despite a probable random character of the Time Curve it gives rise to mutually *correlated* alterations of the locations of different particles or, more generally,—to *global synchronization of random processes of various nature*. At a microlevel this may be related to quantum nonlocality and entanglement, at macrolevel—to universal correlations already observed in the experiments of Shnoll (see, e.g., the review [52]).

Conjecture about random nature of the *Time dynamics* and resulting randomness of the motion of microobjects makes it possible to solve also the problem of concordance between *increments* $\delta T, \delta\mathbf{R}$ of the effective space–time coordinates (33) and *differences* of their final and initial values $\Delta T = T' - T$, $\Delta\mathbf{R} = \mathbf{R}' - \mathbf{R}$[17]. Indeed, for, say, the time coordinate one gets

$$\Delta T := T' - T = (|\mathbf{p} + d\mathbf{p}|^2 - |\mathbf{p}|^2) + (|\mathbf{q} + d\mathbf{q}|^2 - |\mathbf{q}|^2) = 2(\mathbf{p}\cdot d\mathbf{p} + \mathbf{q}\cdot d\mathbf{q}) + (d\mathbf{p}\cdot d\mathbf{p} + d\mathbf{q}\cdot d\mathbf{q}). \quad (43)$$

[17] Generally, these are not necessarily equal due to bilinearity of the induced space–time coordinates with respect to the primary complex "holonomic" coordinates z_μ.

Now under *averaging* the mixed term $dT = 2(\mathbf{p}\cdot d\mathbf{p} + \mathbf{q}\cdot d\mathbf{q})$ vanishes, and the time interval

$$\delta T = d\mathbf{p}\cdot d\mathbf{p} + d\mathbf{q}\cdot d\mathbf{q} \equiv \Delta T \quad (44)$$

at a "physically infinitesimal" scale behaves as a full differential, an actually holonomic entity. The same is true for increments of the *averaged* spatial coordinates $\delta\mathbf{R} \equiv \Delta\mathbf{R}$.

Moreover, property of increment of the time coordinate $\delta T \geq 0$ be positive definite "in average" leads immediately to a natural *kinematical* explanation of the *irreversibility* of physical time. Actually, *any "macroscopic" alteration of the particles' positions (of the state of a system of particles) in the primary complex space necessarily results in an increase of the value of time coordinate of the effective MInkowski space*. Thus, in the algebrodynamical approach *irreversibility of time seems to be of kinematical and statistical nature*, and in the latter respect time resembles the *entropy*-like quantity in the orthodox scheme (if the latter is understood as a *probability measure*).

To conclude, in the context of initially *deterministic* "classical" dynamics there arises an unremovable *uncertainty* and, effectively, *randomness* of evolution of an observable ensemble of micro-objects. This uncertainty is of a global and universal character and is related to conjectural *stochastic* type of alteration of the complex time parameter, to *complex and effectively random nature of physical time itself*. It is noteworthy that numerous problems and perspectives arising under introduction of the notion of *two-dimensional time* have been considered by Sakharov [53];Kechkin and Asadov [45] studied the quantum mechanics with complex time parameter and introduced, in this connection, the notion of different *alteration regimes* of this parameter similar to the above-introduced notion of "evolution curve."

Remarkably, in the capacity of rather unspecified parameter τ of the generating world line $z_\mu(\tau)$ one can (should!) use the principal invariant σ of *complex proper time* (34), with its modulus S corresponding to ordinary Minkowski proper time, and phase α responsible for uncertainty of evolution. This is the only parameter ensuring preservation of both the primary twistor field and the caustic structure (along straight null rays of the generating congruence) (see the proof in [23]). In this sense (despite its accepted name) complex "proper" time acquires the meaning of *universal global time* governing the concordant dynamics of the Universe.

We are now ready to return back to the analysis of quantum interference experiment started in the previous section. Recall that we have undertaken an attempt to relate the wave properties of matter to the conjecture of *dimerous electron* (formed by two pre-elements—duplicons—at the instants of their fusion) and to the geometrical phase (phase of the complex time α) "attached" at each point of the generating world line and lagging along the latter. In the simplest case, assuming linear proportionality of (physically) infinitesimal increments of the module dS and phase $d\alpha$ of complex time,

$$d\alpha = \text{const} \cdot dS, \tag{45}$$

and choosing as the scale factor one half of the inverse Compton length of electron const $= Mc/2\hbar$ (this corresponds to the above-mentioned assumption about the quanta of complex time —"chronon"), one obtains from the fusion condition (42)

$$\Delta\alpha = \frac{Mc}{2\hbar}\Delta\int dS = \frac{\Delta A}{\hbar} = 2\pi N. \tag{46}$$

Essentially, the above formula represents condition for maxima of *classical interference* in the relativistic case. According to it, the phase lag for two "halves"—duplicons under interference are proportional to the path difference, with the Minkowski interval as invariant measure. On the one hand, this is in remarkable correspondence with famous Feynmann representation of the *wave function* $\Psi = R\exp(iA/\hbar)$ whose phase is proportional to the classical action A (for free particle—to the proper time interval). On the other hand, in nonrelativistic approximation decomposing the interval dS over the powers of velocity V/c and taking into account the integrability of zero power term, one obtains as the *condition of quantum interference* the de Broglie relation

$$\Delta\int\frac{dL}{\lambda} = N, \quad \lambda := \frac{h}{Mv}, \tag{47}$$

with *integer* path difference of two duplicons in fractions of the de Broglie wave length λ.

Thus, the phase invariant α seems to be of fundamental physical importance being at the same time a measure of uncertainty of the evolution of microobjects and the measure of their wave properties. The latters have their origin in the peculiarities of primordial complex geometry and do not appeal to the paradigm of wave–particle dualism.

To conclude, we have endeavored to demonstrate the following. Exceptional complex geometry based on the properties of remarkable algebraic structure (biquaternions) when introduced into foundations of physics as the primordial "hidden" geometry of space–time (instead of the habitual Minkowski geometry) results in a quite novel and unexpected picture of the World. As its principal elements one can distinguish identical pre-elements of matter—duplicons—constituents of observable particles (electrons?) as well as uniformly propagating interaction carriers (caustics) and random complex time that predetermines kinematical irreversibility of physical time at macrolevel. However, after 25 years of development of the algebrodynamical field theory on ordinary Minkowski background, "new" complex algebrodynamics is just at the very beginning of its march. We expect that the properties of biquaternion algebra and of associated mathematical structures are rich enough to encode in themselves the most fundamental laws of dynamics and geometry of physical World.

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